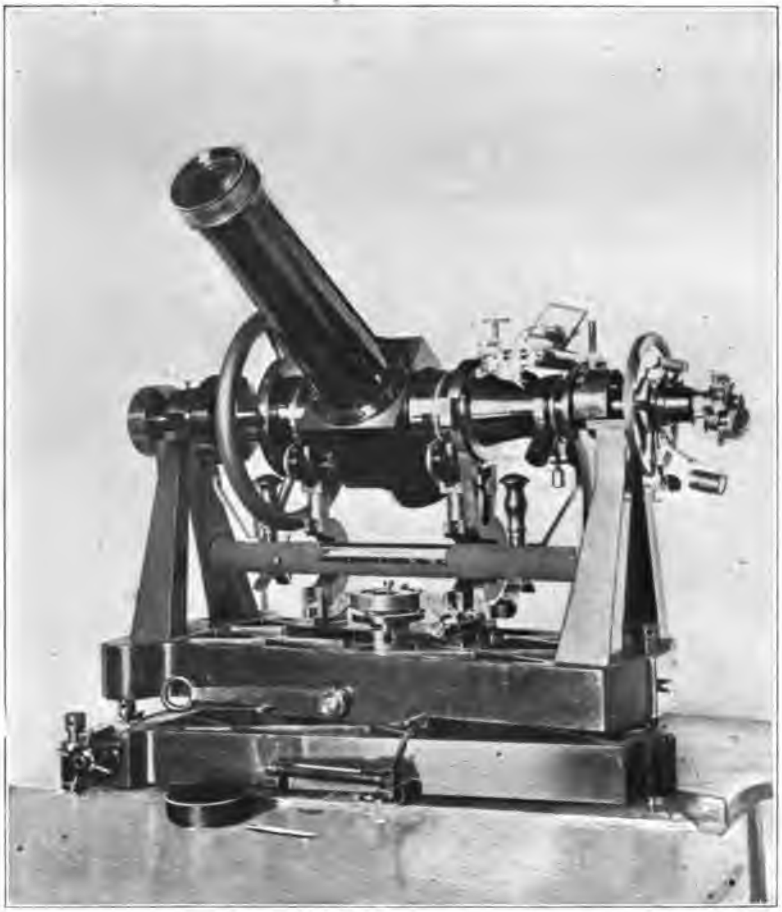


A "BROKEN" TRANSIT.
Length ol axis, 34 inches. Approximate cost, $1200.

A TEXT-BOOK

OF

FIELD ASTRONOMY

FOR ENGINEERS.

BY

GEORGE C. COMSTOCK,

Director of the Washburn Observatory,

Professor of Astronomy in the University of Wisconsin.

SECOND EDITION, REVISED AND ENLARGED.

FIRST THOUSAND.

NEW YORK:

JOHN WILEY & SONS.

LONDON: CHAPMAN & HALL, LIMITED.

1908.

The Scientific Press
Robert Drummond and Company
New York

PREFACE TO THE SECOND EDITION.

THE author has embraced the opportunity afforded by a second edition of this work to introduce into it certain changes which, without modifying the general plan and scope, will, it is hoped, increase its usefulness. For the most part these changes are made in the direction of increased simplicity or of increased precision, and are most conspicuously shown in §§ 19, 23, 32, 37 and 38. The tables at the end of the book have been considerably increased, both in extent and precision, and now suffice for approximate as well as rough determinations of time, latitude and azimuth without the use of an almanac.

June 1, 1908.

PREFACE TO THE FIRST EDITION.

THE present work is not designed for professional students of astronomy, but for another and larger class found in technical colleges. For many years it has been the author's duty to teach to students of engineering the elements of practical astronomy, and the experience thus acquired has gradually produced the unconventional views that find expression in the present text and which, to the author's mind, are justified by the following considerations:

In the engineering curriculum, work in astronomy is a part of a course of technical and professional training of students who have no purpose to become astronomers. Under these circumstances it seems the duty of the instructor to select for presentation those parts of astronomical practice most closely related to the work of the future engineer and, with reference to the narrow limits of time allotted the subject, to keep in the background many collateral matters that are of primary interest and importance to the student of astronomy as a science.

The parts of astronomical practice most pertinent to

engineering instruction seem to the author to be (*a*) Training in the art of numerical computation; (*b*) Training in the accurate use of such typical instruments of precision as the sextant and the theodolite, with special reference to the elimination of their errors from the results of observation; (*c*) Determinations of time, latitude, and azimuth, with portable instruments, as furnishing subject-matter through which *a* and *b* may be conveniently realized. If this work is to be done during the single semester usually allowed for the subject, the time given to its theoretical side, spherical astronomy, must be reduced to the minimum amount compatible with the student's intelligent use of his apparatus and formulæ, and in the present work this pruning of the theoretical side has been carried to an extent that would be unpardonable in the training of an astronomer, but which appears necessary and proper in this case.

Since many engineering students acquire from the mathematical curriculum little or no knowledge of spherical trigonometry and its numerical applications, the first chapter of the work is devoted to a brief presentation of the elements of this subject with special reference to its astronomical uses and to the student's acquisition of good habits in the conduct of numerical work. The astronomical problems presented in the following chapters are those that have been indicated by experience as best adapted to the author's own pupils, and while many of the methods given for their solution are not contained in the current text-books, in every case these are either methods in use in the best geodetic surveys,

or such as have been repeatedly tested with students and found well suited to their use. These methods are classified in the text as rough, approximate, and precise, with respect to their precision and the corresponding amount of time and labor required for their application, and the student is advised not to use the refined and laborious methods when only a rough result is required.

As a rule, in the development of formulæ no attempt has been made to deal with the general case when the solution of a particular case would suffice for the problem in hand; e.g., the earth's compression is ignored in treating of the effect of parallax, since its influence is vanishingly small in the great majority of cases that the student will ever encounter, and cases in which this influence is of sensible amount should be avoided by the instructor. A more serious omission, but one required by the general plan of the work, is found in the theory of the transit instrument, Chapter IX, where broken transits, thread intervals, curvature of a star's apparent path, flexure, etc., are passed by without treatment or even suggestion. They are not required for the beginnings of work with a transit instrument, and therefore constitute a part of more advanced study than is here contemplated. As a partial guide to such study there is given upon a subsequent page a list of references to works that may be consulted with profit by the student who seeks a more complete knowledge of the processes of practical astronomy.

The adopted notation follows, with only slight deviations, that of Chauvenet, to whose elaborate treatise

upon Spherical and Practical Astronomy the author is under obligations that are common to every present-day writer upon those subjects. His thanks are also due to many of his former pupils, and in particular to Dr. S. D. Townley and Dr. Joel Stebbins, who have read and criticised portions of his manuscript.

TABLE OF SYMBOLS.

THE following table contains a brief explanation of the principal symbols employed in the text, with references to the page at which they are respectively defined. There are omitted from the table a considerable number of symbols employed only in immediate connection with their definition.

		Mathematical.
Σ	p. 154	Summation symbol.
[ ]	11	The enclosed number is a logarithm.
		Coordinates, etc.
h	p. 26	Altitude.
z	25	Zenith distance. Complement of h.
A	26	Azimuth, reckoned from south.
a_0, δ_0	65	Azimuth, reckoned from north.
t	26	Hour angle.
δ	26	Declination.
α	26	Right ascension.
ϕ	29	Latitude.
λ	42	Longitude.
S	52	Sun's semi-diameter.
P	53	Horizontal parallax.
R, R'	58	Refraction, etc.
p	70	Polar distance, $=90^\circ-\delta$.
		Time.
θ	p. 30	Sidereal time.
M	41	Mean solar time.
T	44	Time shown by a chronometer, whether right or wrong.
ΔT	44	Chronometer correction.
ρ	45	Chronometer rate.
E	40	Equation of time.
V	40	Date of conjunction, mean sun with vernal equinox.

Q	p.	42	Sidereal time of mean noon, as given in almanac.
Q_1		43	Sidereal time of mean noon reduced to the local meridian.

Rough and Approximate Determinations.

a_0	p.	71	Tabular difference of azimuth, Polaris and north pole.
b_0		71	Tabular difference of altitude, Polaris and north pole.
F, f		71	Factors to transform a_0 and b_0 into their true local values,
H		87	A horizontal angle.
Y, D		72	Auxiliaries for finding hour angle of Polaris.
U		91	An approximate value of the chronometer correction, ΔT, referred to sidereal time.
A_0			An approximate azimuth of a mark.
x, y		91	Corrections to transform U and A_0 into true values.
C		92	A coefficient, equals $\frac{dA}{dt}$.
τ		92	An hour angle.

Instruments.

R, r	p.	118	Circle or micrometer readings.
d		100	Value of half a level division.
k		159	Value of one revolution of a micrometer.
γ		118	Deviation of a vertical axis from the true vertical.
b', b''		114	Components of γ parallel and perpendicular to horizontal axis.
i		118	Deviation, from 90°, of angle between axes of theodolite.
w		120	Complement of spherical angle at zenith between horizontal axis and line of sight.
a		171	Deviation of horizontal axis from true east and west.
b		171	Deviation of horizontal axis from true level.
c		171	Collimation constant.
A, B, C		173	Mayer's transit factors.
C_m		162	Calibration correction to micrometer-screw.

Precise Determinations.

L	p.	144	Correction to equal altitudes. (Time.)
ΔR_0		148	Correction to equal altitudes. (Azimuth.)
g, k		149	Auxiliaries used in computation of azimuths.
s		160	Auxiliary used in computation of differential refraction.
f		83	Auxiliary used in computation of reduction to meridian.
σ		189	Auxiliary collimation coefficient.

TABLE OF CONTENTS.

CHAPTER VI.

FIELD ASTRONOMY.

CHAPTER I.

INTRODUCTORY.

1. **Sphèrical Trigonometry.**—Any three points on the surface of a sphere determine a spherical triangle, whose sides are the arcs of great circles joining these points, and whose angles are the spherical angles included between these arcs; e.g., on the surface of the earth, assumed to be spherical in shape, the north pole, the city of St. Louis, and the borough of Greenwich, England, are three points making a spherical triangle, two of whose sides are the arcs of meridians joining St. Louis and Greenwich to the pole; the third side being the arc of a great circle connecting St. Louis and Greenwich, and measuring by its length the distance of one place from the other. The spherical angle at the pole between the two meridians is the longitude of St. Louis, while the angle at St. Louis between its meridian and the third side of the triangle represents the direction of Greenwich from St. Louis, a certain number of degrees east of north. The particular number of degrees in this angle is to be found by solving

the triangle, i.e., determining the magnitude of its unknown parts by means of the known parts, and in this case we may suppose these known parts to be the difference of longitude between the two places, and the distance of each place from the north pole, i.e., the complement of its latitude.

The formulæ required for the solution of a spherical triangle are best derived by the methods of analytical geometry, and in Fig. 1 we assume a spherical triangle,

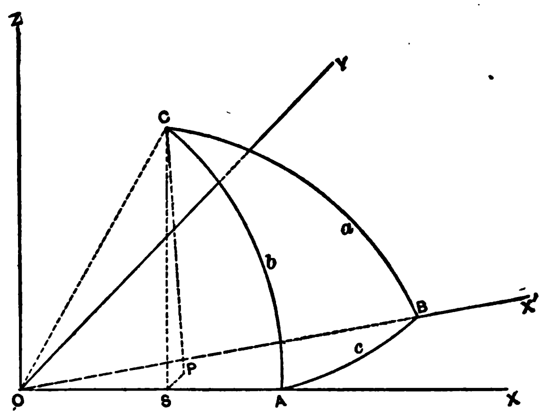


FIG. 1.

ABC, situated on the surface of a sphere whose centre is at *O*, and we adopt *O* as the origin of a system of rectangular coordinates, in which the axis *OX* passes through the vertex, *A*, of the triangle, *OY* lies in the plane *AOB*, and *OZ* is perpendicular to that plane. From the vertex *C* let fall upon the plane *OAB* the perpendicular *CP*, and from *P* draw *PS* perpendicular to *OX* and join the points *C*, *S*, thus obtaining the right-angled plane triangle *CPS*.

The lines OS, SP, PC are respectively the x, y, and z coordinates of the point C, and OC, which we shall represent by the symbol r, is the radius of the sphere.

It is evident from the construction that the points O, S, A, and C all lie in the same plane. Also, O, S, A, B, and P lie in another plane, and the angle between these two planes is measured both by the spherical angle BAC and by the plane angle CSP, and these angles must therefore be equal each to the other. We may now express the coordinates of the point C in terms of the sides, a, b, c, and angles, A, B, C, of the spherical triangle as follows:

$$\begin{aligned} OS &= x = r \cos b, \\ SP &= y = r \sin b \cos A, \\ PC &= z = r \sin b \sin A. \end{aligned} \tag{1}$$

If the axis of x, instead of passing through A, had been made to pass through B, as is shown by the broken line OX', the axis of Z remaining unchanged, we should have had for the coordinates of C in this system,

$$\begin{aligned} x' &= \quad r \cos a, \\ y' &= -r \sin a \cos B, \\ z' &= \quad r \sin a \sin B. \end{aligned} \tag{2}$$

For the sake of simplicity each angle of the triangle ABC has been made less than 90°, and the point P, therefore, falls between the axes OX, OX', thus giving y and y' opposite signs, as shown above.

It is evident from the figure that the relations between x, x', y, y', are those furnished by the formulæ for the

transformation of coordinates in a plane, when the origin remains unchanged and the axes are revolved through an angle, which in this case is measured by the side c of the spherical triangle. We have, therefore,

$$\begin{aligned} z' &= z, \\ y' &= y \cos c - x \sin c, \\ x' &= y \sin c + x \cos c; \end{aligned} \tag{3}$$

and introducing into these equations the values of the coordinates above determined and dividing through by r, we obtain the following relations among the sides and angles of the triangle:

$$\begin{aligned} \sin a \sin B &= \sin b \sin A, \\ \sin a \cos B &= \cos b \sin c - \sin b \cos c \cos A, \\ \cos a &= \cos b \cos c + \sin b \sin c \cos A. \end{aligned} \tag{4}$$

These are the fundamental equations of spherical trigonometry and hold true not only for the particular triangle for which they have been derived, but for every spherical triangle, whatever its shape or size.

2. **Numerical Applications of Equations 4.**—We proceed to apply these equations to the logarithmic solution of the triangle above described, premising that in this solution the signs of all the trigonometric functions must be carefully heeded, since upon them depend the quadrants, first, second, third, or fourth, in which the unknown parts of the triangle are to be found. In this connection we shall reserve the signs + and − for natural numbers and place after a logarithm the letter n whenever the number corresponding to the logarithm is negative. The student

should accustom himself to this practice, since it is the one in general use.

The assumed data of the problem are:

Angular distance, Greenwich to Pole....	$b = 38°.5$,
Angular distance, St. Louis to Pole.....	$c = 51°.4$.
Spherical angle at North Pole..........	$A = 90°.4$

and these data we treat as follows:

SOLUTION.

Logarithms.	Numbers.	Logarithms.
$\sin A = 0.000$	$\cos b \sin c = +0.613$	$\sin a \sin B = 9.794$
$\sin b = 9.794$	$\sin b \cos c \cos A = -0.003$	9.852
$\cos A = 7.844n$		$\sin a \cos B = 9.789$
$\cos c = 9.795$	$\cos b \cos c = +0.489$	$B = 45°.3$
$\sin b \cos A = 7.638n$	$\sin b \sin c \cos A = -0.003$	$\sin a = 9.942$
$\cos b = 9.894$		
$\sin c = 9.893$	$\log \cos a = 9.686$	$a = 61°.0$*

In the solution printed above, the student should examine the orderly manner of the arrangement. Each number is labelled to show what it is, and from these labels we see that the first column contains the logarithms of the several trigonometric functions that appear in the second members of Equations 4. The second column contains natural numbers representing the values of the several terms contained in these second members. These are obtained by adding the proper logarithms shown in the first column, and looking out the corresponding numbers in the tables. An expert computer will do this work "in his head" without writing down a figure that is not shown in the printed solution.

At the bottom of the second column is given log cos a, obtained by looking out the logarithm of the sum of the two numbers that stand just above it. This sum being positive shows that the side a lies in either the first or

fourth quadrant, but it alone cannot decide between these two possibilities. We must now have recourse to the third column, which gives the logarithms of the products, $\sin a \sin B$ and $\sin a \cos B$, as derived from the first and second columns, and indicates that these products are positive quantities, since no n is appended to either of the logarithms. The products being positive, the factors $\sin a$, $\sin B$, and $\cos B$ must all have like signs, and assuming, temporarily, that $\sin a$ is a positive quantity we find that B must lie in the first quadrant, since $\sin B$ and $\cos B$ are positive numbers. To obtain its numerical value we divide $\sin a \sin B$ by $\sin a \cos B$ (subtract mentally the corresponding logarithms) and find, as the result of the division, $\log \tan B = 0.009$. This furnishes the value of B given in the solution, and fixes as the direction of Greenwich from St. Louis, N. 45°.6 E.

Now, looking up in the logarithmic tables the value of $\sin B$ ($\log \sin B = 9.854$), and dividing it out from $\sin a \sin B$, we obtain the value, 9.940, given in the solution for $\sin a$. This number might equally have been obtained by looking up in the tables the value of $\cos B$ ($= 9.845$) and dividing it out from $\sin a \cos B$, and with reference to this double possibility the label for the line between $\sin a \sin B$ and $\sin a \cos B$ is omitted, it being understood that either $\sin B$ or $\cos B$, whichever is the greater of the two, will be entered here and used in the proper manner to obtain $\sin a$. The value of $\log \cos a$ is given in the middle column, and both $\sin a$ and $\cos a$ being positive numbers, a is to be taken in the first quadrant. The agreement between the numerical values of a

furnished by the sine and cosine, is a check upon the accuracy of the computation, and an asterisk or check-mark is placed after the value of a to show that this check has been applied and found satisfactory.

In determining the quadrant of B, sin a was assumed to be a positive number. It might equally well have been assumed a negative number, which would have made sin B and cos B both negative, and would have furnished as the solution of the triangle $B = 225°.6$, $a = 299°.1$. This is also a correct result, for if we travel from St. Louis in the direction N. 225°.6 E., over an arc of a great circle 299°.1 long, we shall find Greenwich at the end of it. The first solution represents the least distance, the second solution the greatest distance, on the surface of the sphere, between the two points, and as a matter of convenience it is customary to use the first solution and to assume that sin a is a positive number.

3. Analytical Applications of Equations 4.—Equations 4 suffice for the solution of any triangle in which there are given two sides and the included angle, but they are not immediately applicable when other parts of the triangle are the data of the problem, e.g., when the three sides are given and the angles are required. A large part of spherical trigonometry, therefore, consists in purely analytical transformations of these equations into forms adapted to different data. For the particular case above suggested, a, b, and c given, we find from the last of Equations 4

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}, \tag{5}$$

by means of which the angle A may be computed with the given data, and similar equations may be written for the other angles.

By transformations more tedious than difficult, and involving the introduction of two auxiliary quantities, defined below by Equations 6, we may change Equation 5 into a form more convenient for computation when all three of the angles are to be determined (see any treatise on spherical trigonometry for the analytical processes involved). As a result of these transformations we have the following auxiliaries,

$$s=\tfrac{1}{2}(a+b+c), \qquad k=\pm\sqrt{\frac{\sin s}{\sin(s-a)\,\sin(s-b)\,\sin(s-c)}}, \tag{6}$$

which determine the angles through the relations

$$\begin{aligned} \cot\tfrac{1}{2}A &= k\sin(s-a) \\ \cot\tfrac{1}{2}B &= k\sin(s-b), \\ \cot\tfrac{1}{2}C &= k\sin(s-c). \end{aligned} \tag{7}$$

Right-angled Spherical Triangles.—Since Equations 4 hold true for all spherical triangles, we may apply them to the special case of a triangle right-angled at A, i.e., one in which the angle A equals 90°. We shall then have $\sin A=1$, $\cos A=0$, and with these special values we obtain by substitution the following equations, which should be compared with the corresponding formulæ of plane trigonometry:

From the first equation, $\sin B = \dfrac{\sin b}{\sin a}$.

From first and second equations, $\tan B = \dfrac{\tan b}{\sin c}$. (8)

From second and third equations, $\cos B = \dfrac{\tan c}{\tan a}$.

From the third equation, $\cos a = \cos b \cos c$.

These equations together with those derived in the preceding sections, while far from covering the whole field of spherical trigonometry, will be found sufficient for the purposes of this work.

4. Approximate Formulæ.—In an important class of cases all the preceding formulæ may be greatly simplified for numerical use as follows: It is shown, in treatises on the differential calculus, that the trigonometric functions may be developed in series; e.g.,

$$\begin{aligned} \sin x &= x - \frac{x^3}{6} + \frac{x^5}{120} - \text{etc.}, \\ \tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \text{etc.}, \\ \cos x &= 1 - \frac{x^2}{2} + \frac{x^4}{24} - \text{etc.}, \end{aligned} \tag{9}$$

where x is expressed in radians (one radian $= 57^\circ.3$, $= 3437'.75$, $= 206264''.8$). When the angle x does not exceed a few minutes of arc, x radians is a small fraction, and its powers, x^2, x^3, etc., are still smaller quantities, so that in these series we may suppress all terms save the first, or all terms save the first two, and the error produced by neglecting these terms of a higher order, as

they are called, is approximately measured by the first term thus neglected. For illustration we assume $x=1°$, and turning this into radians find the results shown in the following short table:

	Radians.	Arc.	
x	$=0.0174533+=$	$1°$	
$\frac{1}{2}x^2$	$=0.0001523+=$	$31''.42$	(10)
$\frac{1}{6}x^3$	$=0.0000009+=$	$0''.18$	
$\frac{1}{24}x^4$	$=0.0000000+=$	$0''.00$	

It appears from the value of $\frac{1}{6}x^3$, here given, that if we are prepared to tolerate in our work an error of one part in a million we may, for an arc of $1°$, substitute the arc itself in place of its sine, in any formula where the latter occurs; and similarly (from the value of $\frac{1}{2}x^2$) we may substitute unity in place of the cosine of an arc of $1°$, if we are willing to admit an error of one part in seven thousand. Expressed in arc these errors are as shown in the table, $0''.2$ and $31''.4$ respectively, and with reference to these numbers we may establish the approximate relations: the square of a degree equals a minute; the cube of a degree equals a second; and find readily, from these relations, the square and cube of any small arc, and thus decide whether, in a given case, these quantities may or may not be neglected. For example: if $x=2°$, we find $x^2=4'$, $x^3=8''$, and for any work in which the data can be depended upon to the nearest minute only, we may assume $\sin x=x$, but we cannot assume $\cos x=1$ without sacrificing some of the accuracy contained in the data.

It must be constantly borne in mind that the series given above are expressed in radians, and that when applied numerically, x and its powers must be trans-

formed from arc into radians by dividing by the appropriate factors given above; e.g.,

$$x \text{ (radians)} = \frac{x''}{206264.8} \qquad (11)$$

The divisor given above is numerically equal to the reciprocal of the sine of 1″, and in place of the preceding equation it is customary to write

$$x \text{ (radians)} = x'' \sin 1''. \qquad (12)$$

As these numerical factors are of frequent use, we record here their values:

$$\left.\begin{aligned} \log \sin 1'' &= 4.6855749 - 10, \\ 206264.8 &= [5.3144251]. \end{aligned}\right\} \qquad (13)$$

Observe the peculiar notation of the last line. The brackets indicate that the number placed within them is a logarithm, and the equation asserts that this bracketed number is the logarithm of the first member of the equation. This use of the brackets is very common and should be remembered.

We may apply to the equations of spherical trigonometry the principles here developed, and assuming that the sides, a, b, c, do not much exceed 1°, i.e., that for a triangle on the surface of the earth the vertices of the angles are not more than sixty or seventy miles apart, we shall find that Equations 8 become

$$\sin B = \frac{b}{a}, \quad \cos B = \frac{c}{a}, \quad \tan B = \frac{b}{c}, \quad a^2 = b^2 + c^2.$$

These are the formulæ of plane trigonometry, and indicate that small spherical triangles may be treated as if they were plane.

The use of these approximate relations is not limited to the solution of triangles, but they may be applied to the trigonometric functions of any small angle wherever found, and we shall have frequent occasion to use them in the following pages.

5. **Numerical Computations.**—Engineer and astronomer alike should acquire the art of rapid and correct computation, and as a means to that end there will be found on subsequent pages examples of numerical work which should be studied with reference to their arrangement and the order in which the several processes were executed. Often the order in which this work was done is not the order in which the numbers appear upon the printed page, although their arrangement upon the page always follows exactly the original computation, and in no case is to be regarded as a mere summary of results, picked out and rearranged after the actual ciphering had been performed. For illustration we revert to the example of § 2, and note that $\sin A$ is the first number written in the solution and $\sin b$ stands second. But the second number actually written down in the computation was $\cos A$, instead of $\sin b$, for, having found the place in which to look up $\sin A$, it is more convenient and more economical to look up $\cos A$ at once, while the tables are open at the right place, rather than to turn away for something else and then have again to find the page and place corresponding to the angle A. Having finished

with the required functions of A, $\sin b$ was next looked out and was followed by $\cos b$, although this required the computer to skip two intervening lines of the computation and, temporarily, to leave them blank.

The general principle here observed is: When a table is open at a given place, look up, before leaving it, all that is to be taken from that place. In order to do this it is necessary to block out the computation in advance, and this was done in the case under consideration, every label, from the initial $\sin A$ of the first column to the concluding a of the last column, being written in its appropriate place before a number was set down or the logarithmic table opened. The form of computation thus prearranged is called a schedule, and it is to be strongly urged upon the student as a measure of economy and good practice, that he should draft, at the beginning of each computation, a complete schedule, in which every number to be employed shall be assigned the place most convenient for its use. In general the beginner will not be able to do this without assistance from an instructor, or from models suitably chosen, and for the purposes of the present work the numerous examples contained in the text may be taken as such models.

Some cardinal points in the arrangement of a good schedule are as follows:

(A) Make it short but complete. Do as much of the work "in your head" as can be done without unduly burdening the mind, and write upon paper only the things that are necessary. But all things that are to be written should have places assigned them in the schedule.

No side computations, upon another piece of paper, should be allowed, and the entire work should be so arranged and labelled that a stranger can follow it and tell what has been done.

(B) When the same quantity is to be used several times in a computation ($\sin b$ appears as a factor in three different terms of the preceding example) the schedule should be so arranged that the number need be written only once, e.g., since $\sin b$ is to be multiplied by both $\sin A$ and $\cos A$ it is placed between these numbers in the schedule, and for a similar reason the product $\sin b \cos A$ is placed between $\cos c$ and $\sin c$. In adding the logarithms to form the product $\sin b \sin c \cos A$, cover $\cos b$ with a pencil or penholder and the addition will be as easily made as if the intervening number were not present.

(C) Frequently, several similar computations are to be made with slightly different data, e.g., it may be required to find the direction and distance of half a dozen American cities from Greenwich. A single schedule should then be prepared and the several computations should be carried on simultaneously, in parallel columns, all placed opposite the same schedule; e.g., look out $\sin A$ and $\cos A$ for all six places before proceeding to find $\sin b$ for any of them, etc. In this particular case $\sin b$ and $\cos b$, depending on the latitude of Greenwich, are the same for all the solutions, and instead of writing their values in each column, they should be written upon the edge of a slip of paper and moved along from column to column as needed. As a memorandum for future

reference they should also be written in one column of the computation. Practise this device whenever the same number is to be used in several different places. See §§ 36 and 40 for examples of two computations depending upon a single schedule.

6. The Trigonometric Functions.—There is opened to the inexperienced computer an abundant opportunity for error in looking out from the tables the trigonometric functions of angles not lying in the first quadrant. The best mode of guarding against such errors is the acquisition of fixed habits of procedure, so that the same thing shall always be done in the same way, and to this end the following simple rules may be adopted:

(1) For any odd-numbered quadrant, first, third, etc. Reduce the given angle to the first quadrant by casting out the nines from its tens and hundreds of degrees (add these digits together and repeat the addition until the sum is reduced to a single digit, less than nine), and look up the required function of the reduced arc.

(2) For any even-numbered quadrant, second, fourth, etc. Reduce the angle to the first quadrant, as above, and look out the function complementary to the one given. The algebraic sign of the function is, of course, in all cases determined by the quadrant in which the original angle falls.

See the following applications of these rules:

Quadrant.	Required.		Equivalent.	Process.
2d, Even	cos 144° 29′	=	−sin 54° 29′	1+4=5
3d, Odd	tan 264° 33′	=	+tan 84° 33′	2+6=8
4th, Even	sin 316° 51′	=	−cos 46° 51′	3+1=4
5th, Odd	cot 414° 18′	=	+cot 54° 18′	4+1=5
6th, Even	tan 499° 49′	=	−cot 49° 49′	Reject the 9
etc.	etc.		etc.	etc.

We may readily formulate a corresponding rule for the converse process, of passing from the function to the angle, as follows:

(1) When the arc lies in an odd quadrant. Look out, in the first quadrant, the angle that corresponds to the given function and add to it the required even multiple of 90°, i.e., 0° or 180°.

(2) When the arc lies in an even quadrant. Change the name of the function (for *cos* read *sin*, for *tan* read *cot*, etc.). Look out, in the first quadrant, the corresponding angle and add to it the required odd multiple of 90°, i.e., 90° or 270°.

See the following examples, in which we represent the required angle by z and suppose that there is given the numerical value of its tangent, e.g., log tan $z = 9.654$. The process of looking out in the several quadrants the angle corresponding to this tangent is as follows:

Quadrant.	Use.	Angle.	Add.	Result.
2d, Even	cot	65° 45′	90°	155° 45′
3d, Odd	tan	24° 15′	180°	204° 15′
4th, Even	cot	65° 45′	270°	335° 45′
etc.	etc.	etc.	etc.	etc.

The degrees, minutes, and seconds of the required angle should be obtained from the table, the multiple of 90° added to them, and the final result written down in its proper place, without writing the intermediate steps.

7. Determination of Angles.—In this connection the student will do well to examine the beginning of a table of logarithmic trigonometric functions and observe how difficult it is to interpolate accurately the value of log sin z or log tan z, corresponding to a small angle, e.g.,

$z = 0° \; 33' \; 17''$. The difficulty comes from the rapid variation of the function, large and changing tabular differences. On the other hand, log cos z changes slowly and may be readily and accurately interpolated. If we take the converse case and suppose the logarithmic function to be given and the corresponding angle required, we shall obtain the opposite result. The angle will be accurately determined by the sine or tangent and very poorly determined by the cosine, e.g., log cos $0° \; 33' \; 17'' = 9.99998$ and every angle between $0° \; 29'$ and $0° \; 36'$ has this cosine, thus leaving a possible error of several minutes in the value of the angle determined from this function, while the log sin, if correctly given to five decimal places, will determine the same angle within a small fraction of a second.

In the interest of precision an angle should always be determined from a function that changes rapidly (large tabular differences), while a quantity that is to be found from a given angle is best determined through a function that changes slowly. In the example of § 2, sin a might have been determined through sin B or cos B, and the former was used for this purpose because it varied the more slowly. In cases of this kind, and they are very common, use the function that stands on the right-hand side of the page, in the tables, and subtract it from the larger of the two numbers, sin a sin B or sin a cos B, and it will be then unnecessary to consider whether it is sine or cosine that is employed.

The angle a, in this example, was determined through its tangent (log tan a = log sin a − log cos a), since the

tangent always varies more rapidly than either sine or cosine and should generally be preferred for this purpose. After obtaining a, its sine and cosine were looked out from the tables and compared with the numbers obtained in the solution, for the sake of the "check" thus furnished upon the accuracy of the numerical work. In subsequent pages other checks will be shown, and these should be applied to test the accuracy of numerical work whenever they are available. The mental strain accompanying a long computation is, under the best of circumstances, considerable, and a check properly satisfied serves to relieve this tension and facilitate the subsequent work.

8. Accuracy of Logarithmic Computation.—The example of § 2 was solved with logarithms extending only to three places of decimals, and corresponding to this use of a three-place table the results are given to the nearest tenth of a degree. If it were required to obtain results correct to the nearest minute or nearest second, a greater number of decimals must be employed (four-, five-, or six-place tables). The labor of using these tables increases very rapidly as the number of decimals is increased, and a compromise is always to be made between extra labor on the one hand and limited accuracy on the other.

As the choice of a proper number of decimal places is usually an embarrassing one for the beginner, there is given below for his guidance a formula intended to represent, at least approximately, the limit of error to be expected in the results of computation on account of the inherent imperfections of logarithms (neglected deci-

mals, etc.). The actual error may fall considerably short of this limit or may overstep it a little. It is evident that the limit will be greater for a long computation than for a short one, and if we measure the length of a computation by the number, n, of logarithms that enter into it and represent by m the number of decimal places to which these logarithms are carried, there may be derived from the theory of probabilities the following expression, in minutes of arc, for the limit of probable error:

$$\text{Limit} = 2800' . \sqrt{n} . 10^{-m}.$$

Applying this formula to the example of § 2 we may put $n = 16$, $m = 3$, and find $10'$ as the limit of unavoidable error; corresponding well with the one-tenth of a degree to which the results were carried. If the data were given to the nearest minute and it were required to preserve this degree of accuracy in the results, we should write,

$$1' = 2800' . 4 . 10^{-m},$$

and solving, find $m = 4.0$, i.e., a four-place table is required for this purpose.

Let the student verify by means of the above equations the following precepts:

To obtain	Use
Tenths of degrees	Three-place tables.
Minutes	Four-place tables.
Seconds	Five- or six-place tables.
Tenths of seconds	Seven-place tables.

If the results are to be expressed in linear instead of angular measure, the limit of error must be represented as a fractional part of the quantity, x, that is to be determined, and corresponding to this case we have

$$\text{Limit} = 0.8\, x . \sqrt{n} . 10^{-m}.$$

Corollary. Do not attempt to obtain from a table more than it is capable of furnishing; e.g., do not interpolate hundredths of a degree in the example of § 2, and in connection with linear quantities do not, as a rule, interpolate more than three significant figures from a three-place table, four from a four-place table, etc.

9. Logarithmic Tables.—There exists a great variety of logarithmic tables of different degrees of accuracy, from three to ten places of decimals, and having determined the number of decimal places required in a given computation, the choice among the corresponding tables is largely a matter of personal taste. The beginner, however, will do well to observe the following rules for distinguishing good tables from bad ones:

(A) Wherever the tabular differences exceed 10, a good table should furnish proportional parts, PP, in the margin of each page, so that the logarithms may be interpolated "in the head."

(B) The tables should be accompanied by tables of addition and subtraction logarithms. For an explanation of these, their purpose and use, the student is referred to the tables themselves, but we note here that by their aid the example of § 2 might have been much

more conveniently solved, as is illustrated in a similar problem in § 15.

The most generally useful tables are those of five decimal places, but computers find it to their advantage to have and use at least one table of each kind, from three to six or seven places. In the examples solved in the present work the following tables have been used:

Three-place,	Johnson.	New York.		
Four-place,	Slichter.	New York.	Gauss.	Berlin.
Five-place,	Albrecht.	Berlin.		
Six-place,	Albrecht's	Bremiker.	Berlin.	

As a very useful supplement to the logarithmic tables a slide-rule and the extended multiplication tables of Crelle and Zimmermann are highly esteemed and where extensive computations are to be made much advantage may be derived through the use of computing machines, of which several types are to be had.

CHAPTER II.

COORDINATES.

10. Fundamental Concepts. — For most purposes of practical astronomy the stars may be considered as attached to the sky, i.e., to the blue vault of the heavens, which is technically called the *celestial sphere*, and is regarded as of indefinitely great radius but having the earth at its centre, so that a plane passing through any terrestrial point intersects this sphere in a great circle, and parallel planes passing through any two terrestrial points intersect the sphere in the same great circle.

If the axis about which the earth rotates be produced in each direction, it will intersect the celestial sphere in two points, called respectively the *north* and *south poles*. If a plumb-line be suspended at any place, P, on the earth's surface, and be produced in both directions, it will intersect the celestial sphere, above and below, in the *zenith* and *nadir* of the place. The direction thus determined by the plumb-line is called the *vertical* of the place.

The figure (shape) of the earth is such that the vertical of any place, when produced downward, intersects the rotation axis, and a plane may therefore be passed through

this axis and the vertical. This plane, by its intersection with the celestial sphere, produces a great circle which passes through the poles, the zenith and nadir, and is called the *meridian* of the place, P.

A plane passed through P perpendicular to the direction of the vertical produces by its intersection with the sphere the *horizon* of P. Any plane passing through the vertical is called a *vertical plane* and produces by its intersection with the sphere, a *vertical circle*. That vertical circle whose plane is perpendicular to the meridian is called the *prime vertical*.

With exception of the poles, all of the terms above defined depend upon the direction of the vertical, and as this direction varies from place to place upon the earth's surface each such place has its own meridian, horizon, zenith, etc., while the poles of the celestial sphere are the same for all places.

A plane passed through the centre of the earth perpendicular to the rotation axis produces by its intersection with the earth's surface the *terrestrial equator*, and by its intersection with the celestial sphere it produces the *celestial equator*.

Owing to the motion of the earth in its orbit we see anything within the orbit from different points of view at different seasons of the year, and by the earth's motion the sun is thus made to describe an apparent path among the stars, making the complete circuit of the sky in a year. This path is a great circle intersecting the celestial equator in two points diametrically opposite to each other, and that one of these points through which the

sun passes on or about March 22 of each year, is called the *vernal equinox*.

11. Systems of Coordinates. — Most of the problems of practical astronomy require us to deal with the apparent positions and motions of the heavenly bodies as seen projected against the sky, and for this purpose there are employed several systems of coordinates based upon the concepts above defined, and three of these systems we proceed to consider. These are all systems of polar coordinates having the following characteristics in common:

(1) The origin of each system is at the centre of the celestial sphere.

(2) Each system has a fundamental plane and consists of an angle measured in the fundamental plane; an angle measured perpendicular to the fundamental plane; and a radius vector. The first of these angles is frequently called the horizontal coordinate, and the second the vertical coordinate, of the system. Latitudes and longitudes on the earth furnish such a system of coördinates. The longitude of St. Louis (horizontal coordinate) is measured by an angle lying in the plane of the equator, which is the fundamental plane of this system. The latitude of St. Louis (vertical coordinate) is measured by an angle lying in a plane perpendicular to the equator, and the radius vector of St. Louis is its distance from the centre of the earth, which latter point is taken as the origin of coordinates.

(3) In each system the horizontal coordinate is measured from a fixed direction in the fundamental

plane, called the prime radius, through 360°. The vertical coordinate is measured on each side of the fundamental plane from 0° to 90°.

(4) Those vertical coordinates are called positive that lie upon the same side of the fundamental plane with the zenith of an observer in the northern hemisphere of the earth. Those that lie upon the opposite side of the fundamental plane are negative.

(5) It is frequently convenient to measure a vertical coordinate from the positive half of a line perpendicular to the fundamental plane instead of from the fundamental plane itself, (e.g., in § 2 we take as the vertical coordinate of St. Louis its distance from the pole instead of from the equator). In such cases this coordinate is always positive and is included between the limits 0° and 180°. If h represent any vertical coordinate measured in the manner first described and z be the corresponding coordinate measured in the second way, we shall obviously have the relation, $z = 90° - h$.

The several systems of astronomical coordinates differ among themselves in the following respects:

(*a*) Different fundamental planes for the systems.

(*b*) Different positions of the prime radii in the fundamental planes.

(*c*) Different directions in which the horizontal coordinates increase.

The data which completely define each system of coordinates are given in the following table together with the names of the several coordinates, the letters by which these are usually represented, and the point

of the heavens, called the pole of the system, toward which the positive half of the normal to the fundamental plane is directed. The terms east and west are used in this table with their common meaning, to indicate the direction toward which the horizontal coordinate increases. The letters associated with the several coordinates are conventional symbols that should be committed to memory.

SYSTEMS OF COORDINATES.

System............	I.	II.	III.
Fundamental plane...............	Horizon	Equator	Equator
Prime radius points toward.......	Meridian	Meridian	Vernal Equinox
Horizontal coordinates increase toward......................	West	West	East
Normal points toward...........	Zenith	North Pole	North Pole
Name of horizontal coordinate...	Azimuth $=A$	Hour angle $=t$	Right Ascension $=\alpha$
Name of vertical coordinate......	Altitude $=h$	Declination $=\delta$	Declination $=\delta$

EXERCISES.—Let the student define in his own language the several quantities above represented by the letters A, t, a, h, and δ.

1. What is the azimuth of the north pole?

2. What do the hour angle and altitude of the zenith respectively equal?

3. What are the azimuths of the prime vertical?

4. What are the declinations of the points in which the horizon cuts the prime vertical?

5. Does δ in the second system differ in any way from δ in the third system?

The directions of the prime radius as above defined for systems I and II, are ambiguous, since the meridian cuts the fundamental plane of each of those systems in two points. Either of these points may be used to determine the direction of the prime radius, but in general that one is to be employed which lies south of the zenith.

Let the student show the relation between the coordinates furnished in System I by adopting each of the possible positions for the prime radius.

12. Uses of the Three Systems.—It is well to consider here, very briefly, the reasons for using more than one system of coordinates, and the relative advantages and disadvantages of these systems.

The coordinates of System I are well adapted to observation with portable instruments, e.g., an engineer's transit, since the horizon is more easily identified with such an instrument than is any other reference plane, and the circles of the instrument may be made to read, directly, altitudes and azimuths. The horizon has been defined by reference to the direction of a plumb-line, but in practice a spirit-level, or the level surface of a liquid at rest, are more frequently used to determine its position.

System I possesses the disadvantage that, through the earth's rotation about its axis, both the altitude and azimuth of a star are constantly changing in a complicated manner, and in this respect System II possesses a marked advantage. Since the normal to its fundamental plane coincides with the earth's axis, rotation about this axis has no effect upon the vertical coordinates, declinations, which remain unchanged, while the horizontal coordinates, hour angles, increase uniformly with the time, 15° per hour, and are therefore easily taken into account and measured by means of a clock.

Suppose a watch to have its dial divided into twenty-four hours, instead of the customary twelve. If this watch be held with its dial parallel to the plane of the equator, the hour hand, in its motion around

the dial, will follow and keep up with the sun as it moves across the sky. If the watch be turned in its own plane until the hour hand points toward the sun, the time indicated upon the dial by this hand will be approximately the hour angle of the sun, and the zero of the dial will point toward the meridian, i.e., south.

Let the student compare the ideal case above considered with the following rough rule sometimes given for determining the direction of the meridian by means of a watch with an ordinary, twelve-hour, dial: Hold the watch with its dial as nearly parallel to the plane of the equator as can be estimated. (See § 13 for the position of this plane.) Revolve the watch in this plane until the hour hand points toward the sun, and the south half of the meridian will then cut the dial midway between the hour hand and the figure XII.

A further advantage is gained in the third system of coordinates, since here the prime radius shares in the apparent rotation of the celestial sphere about the earth's axis, and both the horizontal and vertical coordinates are therefore unaffected by this motion. Instruments have been devised for the measurement of the coordinates in each of these systems, but we shall be mainly concerned with those that relate to the first system, and shall consider System III as employed chiefly to furnish a set of coordinates independent of the earth's rotation and of the particular place upon the earth at which the observer chances to be. These features make it suited to furnish a permanent record of a star's position in the sky, and it is so used in the American Ephemeris (see § 21) and other nautical almanacs, where there may be found, tabulated, the right ascensions and declinations of the sun, moon, planets, and several hundred of the brighter stars.

13. Relations between the Systems of Coordinates.— A problem of frequent recurrence is the transformation of the coordinates of a star from one system to another;

indeed most of the problems of spherical astronomy are, analytically, nothing more than cases of such transformation, and as an introduction to these problems we shall examine the relative positions of the fundamental planes and prime radii of the several systems.

The plane of the equator intersects the plane of the horizon in the *east and west* line, and the angle between the two planes is called the *colatitude*, since it is the complement of the geographical latitude of the place

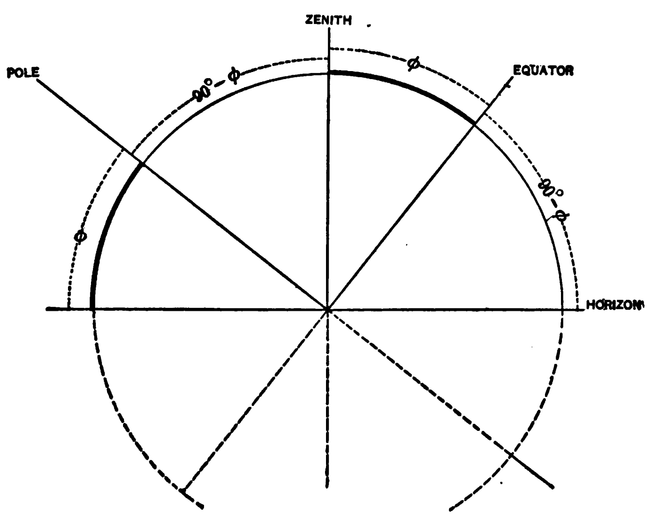


FIG. 2.

to which the horizon belongs. The *latitude* is commonly defined as the angular distance of any place from the equator, but more precisely the latitude is the angle which the vertical of the given place makes with the plane of the equator. From Fig. 2, which represents a meridian section of the earth with the several lines

and planes passed through its centre, it is apparent that the latitude, ϕ, equals the declination of the zenith and also equals the altitude of the pole. The angular distance of the zenith from the pole is equal to the co-latitude, $90° - \phi$.

The second and third systems of coordinates have the same fundamental plane, and their relation to each other is therefore determined by the angle, θ, between their prime radii. Since one of these prime radii is directed toward a fixed point of the heavens, while the other lies in a meridian of the rotating earth, it is evident that the angle θ is continuously and uniformly variable, at the rate of 360° in twenty-four hours. Methods of determining, for any instant, the value of this angle, which is called the *sidereal time*, will be given hereafter. For the present we note that θ may be regarded as the horizontal coordinate of the vernal equinox in the second system, or as the horizontal coordinate of the meridian in the third system, and correspondingly we may define the sidereal time as either the hour angle of the vernal equinox or the right ascension of the meridian.

14. Transformation of Coordinates.—The transformation of coordinates from the first to the second system is conveniently made by means of the "astronomical triangle," i.e., the spherical triangle formed by the zenith, the pole, and the star whose coordinates are to be transformed. In Fig. 3 this triangle is marked by the letters *PZS*, *P* indicating the position of the celestial pole; *Z*, the observer's zenith; and *S*, the apparent place of the star as seen against the sky. Imag-

ine the triangle projected against the sky and the three points to be visible in their true positions.

PZ is an arc of a great circle passing through the pole and zenith and must therefore be a part of the observer's celestial meridian, and in Fig. 2 we have already seen that this arc of the meridian is equal in length to the complement of the latitude, $90° - \phi$. The broken line

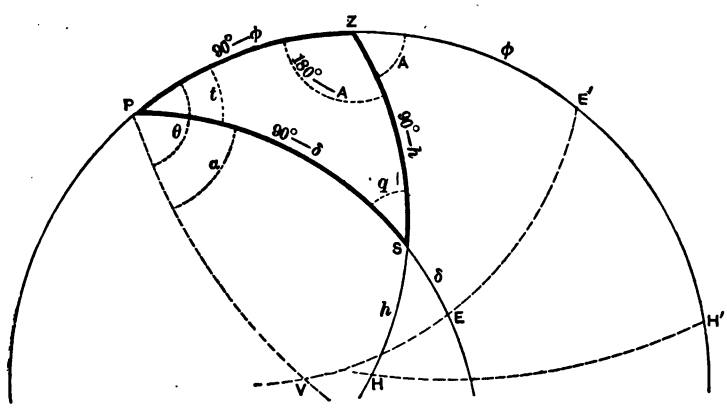


Fig. 3.—The Astronomical Triangle.

HH' in the figure, is an arc of a great circle, every part of which is 90° distant from *Z*. But the great circle 90° distant from the zenith is the horizon, and the arc *HS* that measures the distance of *S* from *HH'* must be the star's altitude, *h*, and the side *SZ* of the astronomical triangle, being the complement of this arc, is equal to $90° - h$. In like manner, *EE'*, drawn 90° distant from *P*, is an arc of the celestial equator; the arc *SE*, that measures the distance of *S* from *EE'*, is the star's declination, δ, and the side *PS* of the triangle equals $90° - \delta$.

The star's hour angle, i.e., horizontal coordinate lying in the equator, is measured by the arc EE', and this arc, by a theorem of spherical geometry, is numerically equal to the spherical angle, t, included between EP and $E'P$, which is therefore the star's hour angle. In like manner the spherical angle SZE' is shown to be equal to the star's azimuth, A, and the angle SZP of the astronomical triangle is equal to $180° - A$. The third angle of the triangle, marked q in the figure, is called the parallactic angle.

To apply to the astronomical triangle the fundamental formulæ of spherical trigonometry derived in § 1, we replace the general symbols used in Equations 4 by the particular values which they have in the astronomical triangle, as follows:

$$\begin{aligned} a &= 90° - h & A &= t \\ b &= 90° - \delta & B &= 180° - A \\ c &= 90° - \phi \end{aligned}$$

and introducing these values into Equations 4 we obtain the required formulæ for transforming altitudes and azimuths into declinations and hour angles, as follows:

$$\begin{aligned} \cos h \sin A &= +\cos \delta \sin t, \\ \cos h \cos A &= -\cos \phi \sin \delta + \sin \phi \cos \delta \cos t, \\ \sin h &= +\sin \phi \sin \delta + \cos \phi \cos \delta \cos t. \end{aligned} \qquad (14)$$

The transformation formulæ between the second and third systems are much simpler. In Fig. 3 if V represent the position of the vernal equinox, we shall have the arc VE', or the corresponding spherical angle at P, equal to the sidereal time, θ, since the sidereal time is the

hour angle of the vernal equinox. Similarly the arc VE and its corresponding angle at P are equal to the right ascension of the star, and from the figure we then obtain the required relations,

$$\alpha + t = \theta, \qquad \delta = \delta. \quad . \quad . \quad . \quad . (14a)$$

The transformation between the first and third systems is best made through the second system; i.e., by using both groups of formulæ 14 and 14*a*.

15. Problem in Transformation of Coordinates. — At the sidereal time $13^h\ 22^m\ 49^s.3$ the altitude and azimuth of a star were measured at a place in latitude $43°\ 4'\ 36''$, as follows: $h = 61°\ 19'\ 36''$, $A = 253°\ 9'\ 42''$. Required the right ascension and declination of the star.

The required transformation formulæ may be obtained from the astronomical triangle in the same manner as Equations 14, and are:

$$\left.\begin{aligned} \cos\delta \sin t &= +\cos h \sin A, \\ \cos\delta \cos t &= +\cos h \cos A \sin\phi + \sin h \cos\phi, \\ \sin\delta &= -\cos h \cos A \cos\phi + \sin h \sin\phi \\ \alpha &= \theta - t. \end{aligned}\right\} \quad (15)$$

Note that θ, which occurs only in the last equation, is expressed in hours, minutes, and seconds of time, and that it is customary to express both α and t in these units, $15° = 1^h$, etc.

A convenient form for the numerical operations involved in solving these equations is given below, and the student should trace it through, verifying each number and ascertaining why the work is arranged as it is.

Compare and contrast this solution with the one contained in § 2. The difference of arrangement is largely due to the introduction here of addition and subtraction logarithms. These are indicated in the schedule by the words Add, Subtract, and it is to be especially borne in mind that the addition indicated by the word Add requires a subtraction logarithm when one of the given terms is itself a negative quantity, etc. The schedule shows the algebraic operation required by the formula, but the arithmetical character of the operation is altered by the presence of an odd number of negative signs.

SOLUTION.

$\sin A$	9.98097*n*	$\sin h \cos \phi$	9.80676
$\cos h$	9.68108	$\cos h \cos A \sin \phi$	8.97740*n*
$\cos A$	9.46191*n*	Add	0.75974
$\sin \phi$	9.83441	$\cos \delta \cos t$	9.73714
$\sin h$	9.94318	(See p. 6)	9.88380
$\cos h \cos A$	9.14299*n*	$\cos \delta \sin t$	9.66205*n*
$\cos \phi$	9.86358	t	$319° 55' 44''$
		$\cos \delta$	9.85334
$\sin h \sin \phi$	9.77759	t (time)	$21^h 19^m 42^s.9$
$\cos h \cos A \cos \phi$	9.00657*n*	θ	$13^h 22^m 49^s.3$
Subtract	0.06797	a	$16^h 3^m 6^s.4$
$\sin \delta$	9.84556	δ	$44° 29' 12''$ *

The declination, δ, is obtained both from $\sin \delta$ and $\cos \delta$, and the agreement of the two values is a 'check' upon the accuracy of the work. The * indicates that this check has been applied.

CHAPTER III.

TIME.

16. In astronomical practice, time is measured by watches and clocks that differ in no essential respect from those in common use, but in addition to the common system of time reckoning, astronomers employ several others, of which we shall have to consider the following:

Sidereal Time, already referred to in § 13.

True Solar Time, which is frequently called Apparent Solar Time.

Mean Solar Time, which is the common system of every-day life.

These three systems possess the following features in common: In each system that common phrase "the time of day" means the hour angle of a particular point in the heavens, which we shall call the zero point of the system. The unit of time is called a *day* and is, in every case, the interval between consecutive returns of the zero point to a given meridian; i.e., consecutive transits of a given meridian past the same zero point. This unit is subdivided into aliquot parts called hours, minutes, and seconds. Each day begins at the instant when the zero point is on the meridian; i.e., on the upper half of

the meridian (noon) in astronomical practice, on the lower half of the meridian (midnight) in civil affairs. In astronomical practice the hours from the beginning of the day are reckoned consecutively, from 0 to 24; in civil practice from 0 to 12, and then repeated to 12 again, with the distinguishing symbols A.M. and P.M. In consequence of the different epochs at which the day begins, the astronomical date in the A.M. hours is one day behind the civil date; e.g.,

Civil Time .... May 10, 5^h A.M. equals
Astronomical Time May 9, 17^h.

In the P.M. hours the dates agree.

Since an hour angle must be reckoned from a determinate meridian, this meridian must be specified in order to make "the time" a determinate quantity, and this specification of the meridian should be included in the name assigned to the time; e.g., Local Time denotes the hour angle of the zero point reckoned from the observer's own (local) meridian. Greenwich Time is the hour angle of the zero point reckoned from the meridian of Greenwich. Standard Time is the hour angle of the zero point reckoned from some meridian assumed as standard; e.g., in the United States and Canada the meridians 75°, 90°, 105°, and 120° west of Greenwich are called standard, and Eastern, Central, Mountain, and Pacific Standard Times are hour angles reckoned from these meridians.

A like practice is followed in the use of the term Noon; e.g., Washington Noon is the instant at which the zero point is in the act of crossing the meridian of Washington.

17. **Longitude and Time.**—We have introduced above a reference to the time at different meridians, and we have now to note that since "the time" is defined as an hour angle, it is evident that the number of hours, minutes, and seconds expressing either time or hour angle will depend upon the meridian from which the latter is measured. The difference between the hour angles reckoned from two different meridians will equal the angle between the meridians, i.e., their difference of longitude, so that if T' and T'' represent the times of any event (whether sidereal, mean solar, or true solar time) referred to two different meridians whose difference of longitude is λ, we shall have $T'-T''=\lambda$. It is customary in astronomical practice to express differences of longitude in hours rather than in degrees, since both members of the preceding equation should be given in terms of the same units.

By transposition of one term in the preceding equation we obtain

$$T'=T''+\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and this extremely simple equation indicates that any given time referred to the second meridian may be reduced to the corresponding time of the first meridian by addition of the difference of longitude, where this difference, λ, is to be counted a positive quantity when the second meridian is west of the first. A very common blunder is to omit this reduction to the prime meridian when interpolating from the almanac (see § 21). Beware of it, and note that the hour and minute for which a quan-

tity is required to be interpolated are usually given in the time of some meridian other than that of Greenwich or Washington, for which the almanac is constructed, and must therefore be reduced to one of these standard meridians, by addition of the longitude, before they can serve as the argument for the tabular quantity sought.

18. The Three Time Systems.—The several time systems differ one from another chiefly in respect of their zero points, and these we have now to consider.

Sidereal Time.—As already indicated, the zero point of this system is the vernal equinox, and since this is a point of the heavens whose position with respect to the fixed stars changes very slowly, it measures well their diurnal motion. In colloquial language, "the stars run on sidereal time," and this system is chiefly used in connection with their apparent diurnal motion.

Solar Time.—As their names indicate, both True Solar Time and Mean Solar Time have zero points that depend upon the sun, and before drawing any distinction between the two systems we recall that, owing to the earth's annual motion in its orbit, the sun's position among the stars changes from day to day (we see it from different standpoints). While this change in the sun's position is not an altogether uniform one and takes place in a plane inclined to that of the earth's rotation (ecliptic, and equator), its net result is that in each year the sun makes one entire circuit of the sky, so that any given meridian of the earth, in the course of a year, makes one less transit over the sun than over a star, or over the vernal equinox. The number of solar days in a year is

therefore one less than the number of sidereal days; e.g., for the epoch 1900, (according to Harkness,)

$$\text{One (tropical) year} = 366.242197 \text{ sidereal days}$$
$$= 365^{d}.242197 \text{ solar days.} \quad (17)$$

It appears from this relation that a sidereal unit of time (day, hour, minute) must be shorter than the corresponding solar unit, a relation that we shall have to consider hereafter.

Apparent, or True, Solar Time.—This system has for its zero point the centre of the sun, and the hour angle of the sun's centre at any moment is, therefore, the true solar time. This system is very convenient for use in connection with observations of the sun, but owing to the irregularities in the sun's motion, above noted, apparent solar days, hours, etc., are of variable length, a day in December being nearly a minute longer than one in September. When time is to be kept by an accurately constructed clock or watch such irregularities are intolerable, and for the sake of clocks and watches there is employed for most purposes the third system, viz.,

Mean Solar Time.—In this system the days and other units are of uniform length and equal, respectively, to the mean length of the corresponding units of apparent solar time. The zero point of the system is an imaginary body, called the mean sun, that is supposed to move uniformly along the equator, keeping as nearly in the same right ascension with the true sun as is consistent with perfect uniformity of motion. The mean solar time at any moment is the hour angle of the mean sun

and, numerically, it differs from the corresponding true solar time by the difference between the hour angles, or right ascensions, of the true and mean suns. This difference is called the *equation of time* (the "sun fast" and "sun slow" of the common almanacs and calendars), and its value for each day of the year, at Greenwich noon and at Washington noon, is given in the American Ephemeris (see § 21) and other almanacs.

To change local solar time from one system to the other we have therefore to interpolate the equation of time from the almanac, with the argument the given local time, reduced to the Greenwich or Washington meridian by addition of the longitude, and apply this difference with its proper sign to the given local time. For example, let it be required to find for the meridian of Denver, and for the date May 10, 1905, the local apparent solar time corresponding to the mean solar time, M, given below. The course of the computation is as follows:

λ, Denver west of Greenwich............	6^h	59^m	$47^s.6$	1
M, Denver Mean Solar Time..........	3	5	10 .5	2
Greenwich Mean Solar Time...........	10	4	58 .1	1 + 2
Equation of Time.		+3	44 .2	3
Denver Apparent Solar Time..........	3	8	54 .7	2 + 3

19. Relation of Sideral to Mean Solar Time.—Once in each year the mean sun passes through the vernal equinox and at this instant, which we shall represent by V, mean solar time and sidereal time agree. At any other moment during the year sidereal time will be greater than mean solar time by an amount, Q, equal to the right ascension of the mean sun at that moment,

and the conversion of the one kind of time into the other is therefore solely a matter of finding the corresponding value of Q, i.e.,

$$\theta = M + Q, \tag{18}$$

where θ and M stand respectively for the corresponding sidereal and mean solar time.

Since Q increases by exactly 24^h in one mean solar (tropical) year we may find its rate of increase, S, by dividing 24^h by the number of days in such year (365.2422) and obtain thus

$$\log S = 0.595781, \text{ minutes of time per day,}$$
$$S = 236.555, \text{ seconds of time per day,}$$

and since the motion of the mean sun is entirely uniform we shall have at any time, T, the value of Q given by the relation

$$Q = S(T - V). \tag{19}$$

The quantity V varies slightly from year to year and its value, expressed in days reckoned from the first Greenwich Mean Noon of each year from 1905 to 1930, is given in Table 2. The instant, T, must be similarly expressed for use in Eq. 19. Use the second column of Table 3 to convert ordinary dates into days from the beginning of the year.

For a rough computation "in the head" it is often convenient to use

$$S = 4^m(1 - \tfrac{1}{70}) \qquad V = \text{March } 22.7 \text{ (Greenwich)} \tag{20}$$

The Nautical Almanac gives for each day of the year the value of Q at Greenwich Mean Noon (in the last column of page II of the monthly calendar), and also gives at the end of the almanac, Table III, a table of proportional parts by which to facilitate the interpolation of Q for intermediate times. If M represents any local mean solar time which it is desired to convert into sidereal time and λ be the longitude of the local meridian west of Greenwich, then will $M+\lambda$ be the corresponding Greenwich time and with this argument we obtain immediately from Table III the correction that added to the tabular value of Q for the given date furnishes its value at the given moment.

For illustration of each process above given for finding Q let it be required to find the sidereal time corresponding to Boston mean solar time $9^h\ 19^m\ 26^s.67$ on August 4, 1905, assuming as the longitude of Boston $4^h\ 44^m\ 15^s$ west of Greenwich.

First Method.

	h. m. s.
M	9 19 26.67
λ	4 44 15
$M+\lambda$	14 3 42
T	217.586
V	82.691
$\log (T-V)$	2.129996
$\log S$	0.595781
$S(T-V)$	531.835 minutes
Q	8 51 50.10
θ	18 11 16.77

Second Method.

	h. m. s.
M	9 19 26.67
λ	4 44 15
$M+\lambda$	14 3 42
Greenwich, Q	8 49 31.43
Table III	2 18.60
Local, Q	8 51 50.03
θ	18 11 16.70

The difference in the results here found for M is due to neglected figures in the fourth decimal place of V and T and illustrates the superior precision as well as convenience of the second method. The latter should

always be employed when an almanac for the current year is available, but the first method may render good service in the absence of such an almanac.

The converse problem of transforming sidereal time into mean solar time is best treated as follows: Let M_1 denote an assumed mean solar time not very different from the true value of M corresponding to the given θ and compute, as above, the corresponding values of Q and θ which we represent by Q_1 and θ_1. If θ_1 proves to be different from the given θ, Q_1 will differ from the true Q by the amount that the mean sun changes its right ascension in the interval $\theta-\theta_1$, i.e., the true value of Q will be

$$Q=Q_1+0^s.164(\theta-\theta_1), \qquad (21)$$

when $\theta-\theta_1$ is expressed in sidereal minutes. When values of Q are taken from the Almanac it is customary to adopt the local mean noon as the value of M_1 since in this case the value of M becomes

$$M=(\theta-Q_1)-0^s.164(\theta-Q_1). \qquad (22)$$

The Q_1 thus defined is obtained by adding to the Almanac value of Q a correction taken from Table III with the observer's longitude as argument, and the last term of the expression is taken from a table of multiples of $0^s.164$ given at the end of the Almanac as Table II. In the absence of the Almanac it is better to take the assumed M_1 as near as may be to the true value of M.

Using the Almanac we reconvert the θ found above into the corresponding mean solar time as follows:

Greenwich, Q	8 49 31.43
Table III, λ	46.70
Local Noon, Q_1	8 50 18.13
θ	18 11 16.70
$\theta - Q_1$	9 20 58.57
Table II, $\theta - Q_1$	1 31.90
M	9 19 26.67

Without the aid of an Almanac, we may assume $M_1 = 9^h$ and by a process entirely similar to the *First Method* employed above find the Q_1 shown in the following schedule and determine from it the required M as follows;

M_1	9 0 0
Q_1	8 51 46.80
θ_1	17 51 46.80
θ	18 11 16.70
$0.164(\theta - \theta_1)$	+ 3.19
Q	8 51 49.99
M	9 19 26.71

20. Chronometer Corrections.—As already indicated, in actual practice the measurement of time is made by clocks or chronometers.

A chronometer does not differ essentially from an ordinary watch, and like the latter is designed to show upon its face, at each moment, the mean solar time (or sidereal time) of some definite meridian, e.g., the meridian 90° west of Greenwich. Since the time indicated by such an instrument is seldom correct, the error of the timepiece must usually be taken into account, and in astronomical practice this is done through the equation,

$$\theta = T + \Delta T, \text{ or } \quad M = T + \Delta T, \qquad (24)$$

where T is the time shown by the chronometer (or watch) and ΔT is the *correction* of the chronometer,

i.e., the quantity which must be added, algebraically, to the watch time in order to obtain true time of the given meridian. When the chronometer is too slow T is less than the true time at any moment, and ΔT is therefore positive in this case and negative when the chronometer is too fast. While the symbol ΔT always represents a chronometer correction its numerical value in a given case depends upon the particular use required, i.e., whether the chronometer time is to be reduced to sidereal, or solar, local, or standard time. In the two Equations 24, therefore, ΔT represents quite different quantities, since θ and M are usually different one from the other, and in every case a special memorandum must be made showing whether the given ΔT relates to sidereal, mean solar, or apparent solar time.

If the chronometer gains or loses, it is said to have a *rate* and ΔT will then change from day to day. If we assume a uniform rate, the relation between T and θ becomes

$$\theta = T + \Delta T_0 + \rho(T - T_0), \qquad (25)$$

where the subscript $_0$ denotes the particular value of ΔT belonging to the chronometer time T_0, and ρ is the rate of the chronometer per day or per hour, positive when the chronometer is losing time. The interval $T - T_0$ must be expressed in the same unit as that for which ρ is given, hours for an hourly rate, etc. A similar equation represents the relation between T and M, but ρ and ΔT will be numerically different from the values required in Equation 25.

A *sidereal chronometer*, i.e., one intended to keep sidereal time, differs from a mean solar chronometer only in the more rapid motion of its mechanism, and is in fact an ordinary timepiece for which $\rho = -3^m\ 56^s.5$ per day. Similarly a watch may be regarded as a sidereal timepiece for which $\rho = +10^s$ per hour. A sidereal chronometer is most convenient for use in observations of stars, since their diurnal motion in hour angle is proportional to the lapse of sidereal time, but these observations may perfectly well be made with a watch or other mean solar timepiece, provided this is treated as a sidereal chronometer with a large rate; e.g., if ρ denote the hourly rate of the watch relative to mean solar time, its hourly rate upon sidereal time will be $\rho' = \rho + 10^s$. Use Equation 25 in connection with this value of ρ' to determine from minute to minute the varying value of ΔT.

21. **The Almanac.** — The American Ephemeris and Nautical Almanac is an annual volume issued by the U. S. Navy Department for the use of navigators, astronomers, and others concerned with astronomical data. These data are for the most part quantities that vary from day to day and whose numerical values are given at convenient intervals of Greenwich or Washington solar time, e.g., the E and Q of the preceding sections, and the right ascensions and declinations of the sun, moon, planets, and principal fixed stars. The variations of these quantities are due to many causes, orbital motion, precession, nutation, aberration, etc., that, in general, lie beyond the scope of the present work, but

we shall have frequent occasion, as in §§ 18 and 19, to take from the almanac numerical values of the quantities above indicated, and these values are to be interpolated for some particular instant of time, usually that of an observation in connection with which they are required, as logarithms are interpolated to correspond to some particular value of the argument of the table. Since quantities are tabulated in the almanac for selected instants of Greenwich or Washington time, the time used as the argument for their interpolation must be referred to one of these meridians (see § 17).

For a detailed account of the way in which the almanac is to be used, consult the explanations given at the end of each volume, under the title, Use of the Tables. In addition to those explanations it should be noted that under the heading Fixed Stars, pages 304–399, there are given three separate tables, from the last of which, bearing the subtitle Apparent Places for the Upper Transit at Washington, accurate coordinates of most of the stars may be obtained for use in the reduction of observations. For the remaining stars, five in number and all very near the celestial pole, special provision of this kind is made in the second table, which bears the subtitle Circumpolar Stars. Look here for the coordinates of Polaris. The first table, under the subtitle Mean Places, etc., gives in very compact form, for all stars contained in the other two tables, right ascensions and declinations, together with their Annual Variations, that may be consulted with advantage when only approx-

imate values of these quantities are required, e.g., in the preliminary selection of stars suitable to be observed.

In this connection the second column of the table of Mean Places, entitled *Magnitude*, deserves especial notice, since it furnishes an index to the brightness of the stars, which is an important element in deciding upon their availability for a given instrument. The brightness of each star is represented by a number adapted, upon an arbitrary scale, to that brightness, so that a very bright star is represented by the number 0, one at the limit of naked eye visibility by 6, and intermediate degrees of brightness are represented by the intermediate numbers, carried to tenths of a magnitude. Polaris is of the magnitude 2.2, and is a conspicuous object in even a very small telescope, provided the telescope is properly focussed. In the telescope of an engineer's transit, stars of the magnitude 4.0 or even 5.0 may be readily observed, while with a sextant, under ordinary conditions, the third magnitude may be taken as the limit of availability.

CHAPTER IV.

CORRECTIONS TO OBSERVED COORDINATES.

It has already been pointed out that the problems of spherical astronomy are in great part cases of the transformation of coordinates between systems having a common origin but different axes, and it should be noted that the observed data for these transformations frequently require some correction before they can be introduced into the equations furnished by the astronomical triangle. Aside from errors arising from defective adjustment or other purely instrumental causes, the observed coordinates of a celestial body may require any or all of the following corrections.

22. Dip of the Horizon. — This correction is required when an altitude is to be derived from a measurement of the angle of elevation of a body above the sea horizon. Owing to the spherical shape of the earth the visible sea horizon always lies below the plane of the observer's true horizon, and the amount of this depression might easily be determined from the geometrical conditions involved, were it not that the rays of light coming to the observer from near the horizon are bent by the atmosphere (refraction), in a manner that does not admit

of accurate estimation in any given case, although its average amount is fairly well known. We therefore abstain from any formal investigation of this correction, and expressing by e, in feet, the observer's elevation above the water, we adopt as a sufficient approximation to the observed amount of the depression, either of the following formulæ,

$$D' = \sqrt{e} - \tfrac{1}{10}\sqrt{e}, \qquad D'' = [1.7738]\sqrt{e}. \qquad (26)$$

The values of D given by these equations are expressed in minutes and seconds, respectively, but owing to variations in the amount of the refraction the numerical values furnished in a given case may be in error by several per cent. As a correction D must always be so applied as to diminish the observed elevation above the horizon.

Note that if the depression of the visible horizon be measured with a theodolite or other suitable instrument, Equation 26 will furnish an approximate value of the elevation of the instrument above the water.

23. **Refraction.** — In general the apparent direction of a star is not its true direction from the observer, since the light by which he sees it has been bent from its original course in passing through the earth's atmosphere. The resulting displacement of the star from its true position is called *refraction*, and, like the similar effect noted in the previous section, its analytical treatment presents mathematical and physical problems whose solution must be sought in more advanced works than the present. Some of the results of that solution

which we shall have occasion to use hereafter are as follows: Save at very low altitudes, less than 10°, the refraction does not sensibly change the azimuth of a star, but its whole effect is to increase the altitude, so that every star appears nearer to the zenith than it would appear if there were no refraction. The amount of this displacement depends chiefly upon the star's distance from the zenith, but is also dependent in some measure upon the temperature of the air and its barometric pressure.

If we represent by h' the star's altitude as measured, by h the corresponding true altitude, by t the temperature, in degrees Fahrenheit, of the air surrounding the observer, and by B the barometric pressure in inches, the amount of the refraction in seconds of arc may be obtained from the following formula within half a second for all altitudes greater than 20°,

$$h'-h=R=975''\frac{B}{456+t}\cot h'. \qquad (27)$$

For altitudes less than 20° this formula gives results that are too great and as the star approaches the horizon the error increases rapidly, as may be seen from the following brief table of corrections required to reduce the numbers given by Eq. 27 to the true values of the refraction.

h	Corr.
5°	−68″
10	− 9
15	− 2
20	− 0
25	+ 0

The readings of a mercurial barometer, B', do not

furnish immediately the barometric pressure, B, but require a "reduction to the freezing-point," i.e., a correction to reduce the reading to what it would be if the mercury were at the normal temperature assumed in the theory of the barometer. This reduction may be obtained with sufficient accuracy from the equation

$$B' - B = \frac{B'(T - 29^\circ)}{10\,000}\left(1 - \tfrac{1}{10}\right), \qquad (28)$$

where T is the temperature of the mercury, in degrees Fahr., and the barometer reading and its resulting correction are expressed in inches.

24. **Semi-diameter.**—Observations of the sun or other body presenting a sensible disk are usually made by pointing the instrument at the edge of the body, technically called the limb, and the resulting altitude or azimuth is that of the limb observed, while the data furnished by the almanac relate to the centre of the body. The semi-diameters of the sun, moon, and planets, i.e., the angles subtended at the earth by their respective radii, are given in the almanac at convenient intervals of time, and the interpolated values of these quantities may be used to pass from the observed coordinates of the limb to those of the centre of the body, e.g., the sun. In the case of the altitude or zenith distance we have the very simple relation

$$h' = h'' \pm S, \qquad (29)$$

where S denotes the semi-diameter and h'' and h' are, respectively, the observed and the corrected altitude. The

sign of S depends upon whether the lower or the upper limb was observed.

In the case of an azimuth the relation is more complicated. From the right-angled spherical triangle formed by the zenith, the sun's centre and that point of the limb at which the latter is tangent to a vertical circle (see Fig. 4) we obtain,

$$\sin z \sin (A' - A'') = \sin S, \qquad (30)$$

which determines the correction, $A' - A''$, for difference of azimuth between centre and limb. Since S does not much exceed 15′, we may in most cases assume the arcs to be proportional to their sines and simplify this rigorous equation to the form,

$$A' = A'' \pm S \sec h, \qquad (31)$$

in which the positive sign is to be used for the following and the negative for the preceding limb.

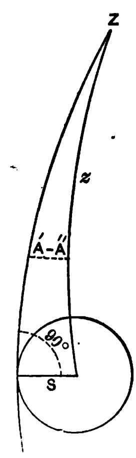


FIG. 4.—Semi-diameter.

25. Parallax. — In the reduction of astronomical observations it is usually necessary to combine the observed coordinates, azimuth, altitude, etc., with data obtained from the almanac, e.g., the right ascension and declination of the body observed. But the origin to which these latter coordinates are referred is the centre of the earth, while the origin for the observed coordinates is

at the eye of the observer, and before combining these heterogeneous data we must reduce them to a common origin, for which we select that used in the almanac.

In Fig. 5 let C represent the centre of the earth, O the observer's position, and P the observed body, at the respective distances ρ and r from C. Neglecting the

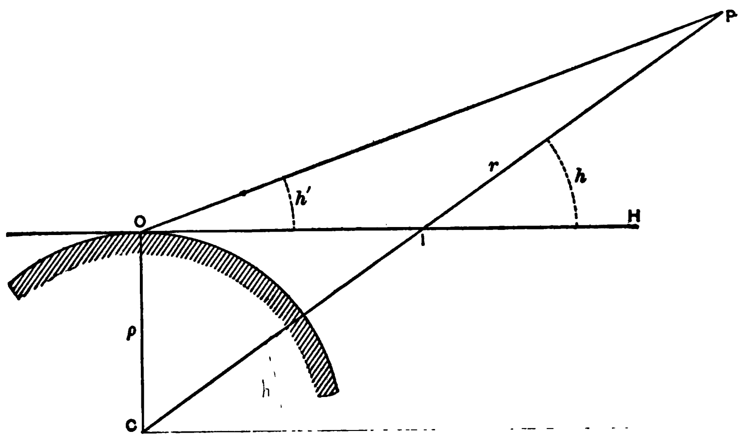


FIG. 5.—Parallax.

earth's compression, i.e., its slight deviation from a truly spherical form, the line OC is the observer's vertical and, therefore, OPC is a vertical plane and marks out upon the celestial sphere a vertical circle, against which the body P will appear projected whether seen from O or C. Its azimuth will, therefore, be the same for the two origins and requires no reduction to the centre of the earth.

The altitude, however, does require such a reduction, and to determine its amount we let OH in Fig. 5 represent the plane of the observer's horizon and obtain as

the observed altitude of P the angle there marked h'. As seen from the centre of the earth the altitude of P will be measured by the angle PIH, marked h in the figure, and from principles of elementary geometry we have

$$h = h' + \angle OPC.$$

This last angle is called the parallax in altitude, and representing it by P we find from the triangle OPC

$$\rho \sin (90^\circ + h') = r \sin P.$$

Since r is always much greater than ρ, P must be a small angle and, applying the principles of § 4, we may write, in place of the preceding equation,

$$P = h - h' = 206265 \frac{\rho}{r} \cos h', \tag{32}$$

which is the required correction to reduce an observed altitude to the corresponding coordinate referred to an origin at the centre of the earth.

For the fixed stars this correction is absolutely insensible, less than $0''.0001$, on account of their great distance from the earth. For the sun and planets it amounts to a few seconds of arc, and in its computation the value of the coefficient, $206265 \frac{\rho}{r}$, should be taken from the almanac, where it is given for each of these bodies and is called their horizontal parallax, since it is the amount of the parallax in altitude when the body is in the horizon, $h' = 0^\circ$. For the sun it is usually sufficient to assume

$8''.8$ as a constant value for its horizontal parallax. The moon's parallax is much greater, about 1°, and the simple analysis given above neglects some factors that are of sensible magnitude in this case, although for ordinary purposes they may be ignored in connection with every other celestial body.

Since the effect of parallax is to make the body appear farther from the zenith than it really is, the corrections for parallax and refraction will always have opposite signs.

26. As an example of such corrections and their proper sequence we give the following reduction of an angle observed between the sun's upper limb and the sea horizon, as seen by an observer at an elevation of 63 feet above the water.

Date	Oct. 6, 1907	Refraction Const.	2.989
Temperature	44° Fahr.	B	1.466
Barometer	29.26 in.	colog $(456+t)$	7.301
$B'-B$	.04	cot h'	0.282
B	29.22	log R	2.038
Observed Angle	27° 43′ 20″	R	109″
		h''	27 33 39

Dip of Horizon		Semi-Diameter	16 2
e	63	h'''	27 17 37
Constant	1.774	Parallax Const.	0.944
log $\sqrt{e}$	0.900	cos h'''	9.949
Dip	472″=	Parallax	8″
Dip	7′ 52″	h	27 17 45
h'	27 35 28		

All computations such as the above may conveniently be made with a slide rule.

In accordance with general custom the symbol log is printed in the above schedule only when necessary to

avoid misunderstanding, as at the bottom of the first column. Usually the figures themselves indicate whether they are logarithms or natural numbers; e.g., the several numbers marked Const. are clearly the logarithms of constant coefficients. For similar reasons of convenience the -10 that strictly should be placed after a logarithm whose characteristic has been increased by 10 is usually left to the imagination.

27. Diurnal Aberration. — There is a very small correction to observed data, arising from the fact that the observer himself is not at rest relative to the stars, but is always in rapid motion toward the east point of his horizon, carried along by the earth in its diurnal rotation. This correction is so small that it may usually be omitted and we therefore abstain from an analytical investigation of its effect, such as may be found in the larger treatises upon spherical astronomy, and note as a result of that investigation that all stars when near the meridian are displaced toward the east point of the horizon through an angular distance equal to $0''.32 \cos \phi$, where ϕ denotes the observer's latitude. As a result of this displacement each star comes a little later to the meridian than it otherwise would come and since the rate of motion of a star when measured in arc of a great circle is proportional to the cosine of its declination, the amount of this retardation, expressed in time, is $0^s.021 \cos \phi \sec \delta$. See the theory of the transit instrument for an example of the application of this correction, and see also the determination of precise azimuths for another case in which diurnal aberration is to be taken into account.

CHAPTER V.

ROUGH DETERMINATIONS OF TIME, LATITUDE, AND AZIMUTH.

28. General Considerations. — For the purposes of field astronomy, which are the only ones contemplated in the present work, the most important astronomical problems relate to the determination of time, latitude, and azimuth.

A time determination implies the making and reducing of astronomical observations which suffice to furnish the correction, ΔT, of a chronometer or other timepiece, and for this purpose we obtain from §§ 15 and 20 the relations

$$\alpha + t = \theta = T + \Delta T, \tag{33}$$

where α and t represent the right ascension and hour angle of any star at the chronometer time T. The student should particularly note that the chronometer is not supposed to be correctly set; T is the time shown by the chronometer regardless of whether that time be right or wrong, since the ΔT fully compensates for any error of this kind. In the case of the sun we have, from the relation between mean and apparent solar time,

$$E + t = M = T + \Delta T, \tag{34}$$

where E denotes the equation of time at the instant T.

Since a and E may be obtained from the almanac, any observation which determines the hour angle of a celestial body at the observed time T will suffice to determine ΔT, and such an observation when properly reduced constitutes a time determination.

An azimuth determination may be required either for fixing the true azimuth of the line joining two terrestrial points, or for determining the relation of a particular instrument to the meridian; e.g., to determine the reading, K, to which the azimuth circle of a theodolite must be set, in order that the line of sight shall point due south. A theodolite is said to be *oriented* when its verniers have been set to read the true azimuth of the object toward which the line of sight is directed, i.e., when $K=0$.

By a latitude determination we mean any set of observations from which a knowledge of the observer's latitude may be obtained.

For each of these determinations, time, azimuth, latitude, many methods have been devised and these differ greatly among themselves with respect to the instrumental equipment and expenditure of time and labor which they require, and with respect to the corresponding degree of accuracy furnished in their results. In any given case a choice must be made among these methods with reference to the required precision of the results and also with reference to convenience and economy in obtaining it. To facilitate this choice the methods to be presented in the following pages are classified as:

(A) *Rough Determinations;* in which there may be

permitted in the final result an error amounting to two minutes of arc or one tenth of a minute of time.

(B) *Approximate Determinations;* in which the final errors ought not to exceed $15''$ and 1^s respectively.

(C) *Accurate Determinations;* in which the required precision is limited only by the capacity of the instrument and of the observer. In the case of a sextant this limit may be placed at $2''$ or $3''$, and for a good engineer's transit at $1''$. We proceed first to consider that class of observations whose advantage consists in economy of time and labor, viz., rough determinations.

29. Latitude. — A determination of any one of the quantities time, latitude, or azimuth is greatly facilitated by a knowledge of one or both of the others, and if all three are unknown, the simplest mode of procedure is to observe the Pole Star as set forth in § 32. But this commonly requires observations by night, which may be inconvenient, and by day the sun is the object most readily available.

From the astronomical triangle, or from Equations 14, it is apparent that when the sun is on the meridian, i.e., when $t=0$, its altitude is a maximum, and if this maximum altitude be measured with a sextant or theodolite it will furnish a latitude determination through the equation, true for noon only,

$$\phi=\delta+z=90^\circ+\delta-h, \tag{35}$$

which may be obtained by inspection from Fig. 2, or analytically from the last of Equations 14. With the instrument employed, follow the sun's motion in altitude

until it begins to diminish, and take the greatest reading obtained as corresponding to the maximum altitude.

This reading, or the altitude, h', derived from it, will require correction for instrumental errors, semi-diameter, etc., as shown in Chapter IV, but the application of these corrections may be abbreviated by interpolating, in minutes of arc, the combined correction for refraction and parallax from Table I, at the end of the book.

This gives, with the observed altitude as argument under the heading R, the amount of the refraction corresponding to an average condition of the atmosphere, viz., Bar. 29.0 in., Temp. 50° Fahr., while under the heading R' is given the combined effect of refraction and parallax for the sun. Use these tables for the reduction of any altitude of sun or star when the required precision is not greater than 10″. This table may be adapted even to extreme atmospheric conditions by increasing R or R' by one per cent for each 0.3 inch that the barometric pressure exceeds 29 inches. Similarly diminish R by one per cent for each 5° that the temperature exceeds 50° Fahr.

The following latitude determination was made by measuring with a sextant and artificial horizon (§ 59) the maximum double altitude of the sun's lower edge (limb) upon a date, Dec. 19, 1898, at which the sun's declination, as furnished by the almanac, was, $\delta = -23° 26'.0$. The reduction of the observation is as follows:

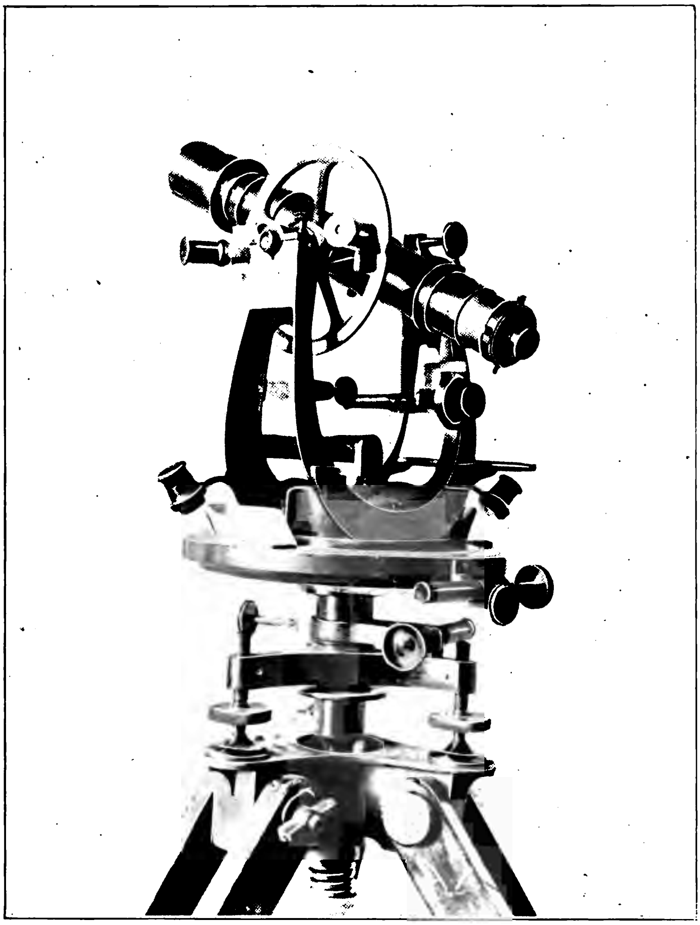
Sextant Reading	57° 44′ 30″
Instrumental Corr.	− 1 50
Corr'd Sextant	57 42 40
h'	28 51.3
Ref.-Par., R'	− 1.6
Semi-diameter	+ 16.3
h	29 6.0
$90° + \delta$	66 34.0
Latitude, ϕ	37 28.0

Make a determination of your own latitude by a similar method.

30. Time and Azimuth from an Observed Altitude.— If the latitude be thus observed at noon, time may be determined with a sextant, and both time and azimuth may be determined with a theodolite by measuring an altitude of the sun when at a considerable distance from the meridian. Any error in the assumed latitude or in the measured altitude will affect both the azimuth and time resulting from the observation, but the influence of these errors may be greatly reduced by observing the sun when near the prime vertical and, for azimuth, when at a low altitude, e.g., 10° or 15°.

When a theodolite is used, there should be at least two observations of altitude made, one Circle R. and the other Circle L., in order to eliminate instrumental errors (see § 50). Observe the edges of the sun, not its center, and correct the results for semi-diameter (§ 24); but if the instrument is provided with stadia threads, this correction may be avoided as follows: Point the telescope at the sun so that the two horizontal threads cut off equal segments from the upper and lower

PLATE I.



An American Engineer's Transit. Diameter of Horizontal Circle 7 inches.
Approximate Cost $350.

[*To face p.* 62.]

edges of the sun, and by turning the slow-motion screw in altitude, keep these segments of equal area as the sun drifts across the field of view, until it reaches a position in which the vertical thread bisects each segment. Record this time to the nearest second, and also record the readings of the four verniers of the instrument. Before reversing the instrument to obtain the second observation, read and record both its levels (azimuth and altitude levels, §§ 48 and 50), and after reversing bring the bubbles back, by means of the levelling-screws, to the position thus recorded. This process eliminates errors of level.

The better class of engineer's transits are usually provided with shade-glasses to moderate the intensity of the sun's light and permit it to be viewed through the telescope. But these glasses are by no means necessary, since an image of the sun and the threads of the instrument may be projected upon a piece of cardboard and be there seen and observed quite as accurately as in the telescope. Pull the eyepiece out, away from the threads, until the latter can no longer be seen distinctly with the eye; then allow the sun to shine through the telescope upon the cardboard held behind the eyepiece, and shift the cardboard toward and from the instrument until a position is found in which the projected images of the threads appear sharp and distinct. Then turn the focussing-screw until the edge of the sun's image also appears well defined and the projected images will be ready for observation. To diminish as far as possible the effect of errors in the measured altitude

and the latitude assumed in the reduction, observe the sun when it is as near as may be due east or west, but its altitude should not be less than 10°.

Reduction of the Observations.—Observations conducted as in the following record will furnish as data the true altitude of the sun's centre and the horizontal angle, H, between the sun's centre and some fixed object whose azimuth is to be determined, both quantities corresponding to one and the same time, T, taken from some watch or other time piece. These data must be supplemented by the latitude, ϕ, supposed to be independently known, and by the sun's declination, δ, and equation of time, E, to be obtained from the almanac. Reference to Fig. 3 will show that δ, h and ϕ are respectively the complements of the three sides of the astronomical triangle, and putting

$$90° - \delta = a$$
$$90° - h = b$$
$$90° - \phi = c$$
$$2s = a + b + c.$$

We find that the solution of the triangle is given by Eqs. 6 and 7, when in place of the general symbols there employed for the angles of the triangle we substitute the particular values which they have in this case, i.e.,

$$k = \pm\sqrt{\frac{\sin s}{\sin(s-a)\sin(s-b)\sin(s-c)}},$$

$$\left.\begin{aligned} \cot \tfrac{1}{2}a_0 &= k \sin(s-a), \\ \cot \tfrac{1}{2}t &= k \sin(s-b), \\ \cot \tfrac{1}{2}q &= k \sin(s-c). \end{aligned}\right\} \qquad (36)$$

The symbol a_0 here introduced to represent the angle of the triangle opposite the side $90° - \delta$, is readily seen from Fig. 3 to be the sun's azimuth reckoned from north toward west, and if we desire to retain the customary mode of reckoning azimuths, from the south, we must write $A = 180° - a_0$. If to A there be added the observed horizontal angle, H, we shall of course find the desired azimuth of the terrestrial point, viz.,

$$A_M = 180° + H - a_0. \tag{37}$$

The angle t is the sun's hour angle at the moment of observation, T, and we have therefore,

$$T + \Delta T = t + E \tag{38}$$

from which to determine the chronometer correction. The third angle of the triangle, q, is introduced only for the sake of the following control which it furnishes upon the reduction. Multiplying together the last three equations of (36) we find,

$$\cot \tfrac{1}{2}a_0 \cot \tfrac{1}{2}t \cot \tfrac{1}{2}q = k \sin s, \tag{39}$$

a relation that must be satisfied within one or two units of the last decimal place if the numerical work of reduction has been correctly performed. Another useful "check" that should be applied midway in the computation is

$$(s - a) + (s - b) + (s - c) = s. \tag{40}$$

The ambiguous sign of k, Eq. 36, covers the two cases of observations made when the sun is west or east

of the meridian. For an A.M. observation use the minus sign, and find a_0, t and q, all negative quantities.

The following illustrative record and reduction relate to observations made at a place whose latitude and longitude are respectively $43° 4.6'$, and $5^h 58^m$ west of Greenwich.

ALTITUDE AND AZIMUTH OF SUN'S CENTRE.

At Station A.—April 16, 1897.

Engineer's Transit, B. Watch No. 6. Observer, C.

Object.	Circle.	Watch.	Vertical Circle.		Horizontal Circle.	
			Ver. A.	Ver. B.	Ver. A.	Ver. B.
		h m s	° ′ ″	′ ″	° ′ ″	′ ″
Sta. B	R.	....	....	....	7 54 10	54 10
Sun	R.	4 12 33	26 26 0	25 50	259 14 0	13 50
Sun	L.	4 15 28	25 57 20	57 0	79 45 0	45 0
Sta. B	L.	....	....	....	187 54 10	54 10

REDUCTION.

δ	10° 28′.0	sin $(s-a)$	9.4300	a_0	100° 30′.4
Vert. Circ.	26 11.5	sin $(s-b)$	9.7157	$180°+H$	288 24.7
R′. Table I_B	−1.8	sin $(s-c)$	9.8726	A_M	187 54.3
h	26 9.7	Sum	9.0183		
ϕ	43 4.6	sin s	9.9982	t (arc)	63° 50′.0
		k^2	0.9799	t (time)	4^h 15^m 20^s
a	79 32.0	k	0.4899	E	−25
b	63 50.3			$t+E$	4 14 55
c	46 55.4	cot $\frac{1}{2}a_0$	9.9199	T	4 13 55
$2s$	190 17.7	cot $\frac{1}{2}t$	0.2056	ΔT	+1 0
s	95 8.8	cot $\frac{1}{2}q$	0.3625		
$s-a$	15 36.8	Sum	0.4880		
$s-b$	31 18.5	k sin s	0.4881*		
$s-c$	48 13.4				
Sum	95 8.7*				

The true azimuth of Station B was known to be $187° 54' 10''$, and a comparison of the watch with standard time furnished as the true value of ΔT, $+57$ seconds. The differences between these results and those found in the preceding solution furnish a fair idea of the precision to be expected in such work.

When time only is to be determined from an observation of the sun the measurement of the angle H is unnecessary, and the reduction of the altitude observation may be made by the method of § 36. Where azimuth only is required both H and h must be observed, and the sun's azimuth may be computed somewhat more conveniently than above, from the third of Eq. 15, written in the form,

$$\text{Cos } A = \frac{\sin\phi \sin h - \sin\delta}{\cos\phi \cos h}, \tag{41}$$

but as no "check" relation accompanies this formula the longer computation given above will often be preferable.

31. Time by Meridian Transits.—If astronomical observations are to be made for any considerable length of time at a given station, as at a university, it will be convenient for many purposes to determine the azimuth of a permanently marked line, at one end of which an instrument can be set up and oriented. If a theodolite be thus mounted and its line of sight brought into the meridian, a time determination may be very simply made by observing the chronometer time of transit of the sun's preceding and following limbs past the vertical thread of the instrument. Since the thread is by supposition in the meridian, the hour angle of the sun at the mean of the observed times, T, is zero and we have

$$\Delta T = \alpha - T \text{ (Sidereal)},$$

or

$$\Delta T = E - T \text{ (Mean solar)}. \tag{42}$$

If the azimuth of the line is well determined, this method may rank as an approximate rather than a rough determination, since under ordinary circumstances there must be an error of nearly 2′ in the orientation of the instrument, to produce an error of 6^s in the chronometer correction. In any case the instrument must be carefully levelled, particularly in the east and west direction, and in the following example the readings of the striding level are employed as a control upon this adjustment.

Observe the slight variation of method here introduced in order to obtain in place of a single observation two observations, one Circle R., and one Circle L.

TRANSITS OF SUN FOR TIME DETERMINATION.

At Station A. April 17, 1897.

Theodolite, F. Watch No. 6. Observer, G.
Instrument oriented on Station B.

Circle.	Ver. A.	Limb.	Watch.	Striding level.	Reduction.
	° ′		h. m. s.		$T = 11^h\ 57^m\ 37^s$
R.	359 30	Pr.	11 55 34	1.4 12.6	$12^h + E = 11\ \ 59\ \ 23$
L.	180 30	Fol.	11 59 40	14.0 0.5	$\Delta T = \ \ +1\ \ 46$

By a comparison with standard time the true ΔT referred to the local meridian was found to be $+1^m\ 47^s$.

The telescope of an engineer's transit is usually capable of showing a first-magnitude star by daylight whenever the sky is clear and blue, and such a star is equally available with the sun for a determination of either latitude or time. In the spring and summer Sirius, by reason of its great brilliancy, is a peculiarly favorable object for such observations (see Table 5

for the approximate right ascension and declination of this and other stars). Even the brightest of these stars is not a conspicuous object by daylight, and is most readily found by placing the telescope in the meridian and at the proper zenith distance, $z = \phi - \delta$, and awaiting its arrival in the field of view. A very slight error of focus in the telescope will render the star invisible, and this adjustment should therefore be carefully made upon a distant terrestrial object before setting the telescope for the star.

32. Orientation by Polaris.—If a rough determination of time, latitude, or azimuth is to be made by night, or if a theodolite is to be oriented as a preparation for other work, observations of the Pole Star by the following method will be found especially convenient, since no almanac is required and no instrumental equipment other than an engineer's transit and a watch. Even the error of the watch need not be known in advance. If Polaris were exactly at the pole of the heavens the instrument might be oriented by pointing directly upon the star and setting the verniers of the horizontal circle to read 180°; and simultaneously the latitude might be determined by measuring the star's altitude, since in this case. $\phi = h$. As Polaris is actually more than a degree distant from the pole this ideal method is inapplicable, but the principles upon which it is based may be applied through the tables at the end of the book. We begin with Table 4, which furnishes the amounts a_0, b_0, by which the azimuth and altitude of Polaris, as seen from a given place at a given time, differ from the corre-

sponding coordinates of the pole, and provision is made through other Tables, 1, 2, 3, for adapting these differences of azimuth and altitude to other times and places. The mathematical basis of the method is as follows:

In the first and third of Eq. 14 let us replace the declination, δ, by the star's polar distance, $p=90^\circ-\delta$, and for the ordinary azimuth, A, let us substitute an azimuth, a_0, reckoned from the north point, i.e., $a=A-180^\circ$.

When the resulting equations,

$$\begin{aligned}\cos h \sin a_0 &= -\sin p \sin t \\ \sin h &= \sin \phi \cos p + \cos \phi \sin p \cos t,\end{aligned} \tag{43}$$

are applied to Polaris we find that p and a_0 are small quantities rarely exceeding 1° or 2°, whose third powers may be neglected without producing error exceeding $1''$ or $2''$, and to this degree of accuracy we may use in their place the simpler forms

$$\begin{aligned}a_0 &= -p \sec h \sin t \\ h &= \phi + p \cos t - \tfrac{1}{2}p^2 \sin^2 t \tan \phi.\end{aligned} \tag{44}$$

The quantities given in Table 4 are respectively the terms of these equations,

$$a_0 = -p \sec h \sin t \qquad\qquad b_0 = p \cos t$$

when p is put equal to $70'.5$ and h is equal to 40°. To adapt these tabular quantities to other values of p and h let us introduce the auxiliary quantities,

$$F = p \div 70'.5 \qquad\qquad f = \sec h \div \sec 40^\circ$$

and substitute for the star's true altitude, h, its apparent altitude, h', corrected for the amount of the refraction, R, and in terms of these find from Eq. 44,

$$\begin{aligned} A &= 180^\circ + Ffa_0 \\ h' - \phi &= Fb_0 + R - \tfrac{1}{2}p^2 \sin^2 t \tan \phi. \end{aligned} \tag{47}$$

The last term in this expression is very small, on account of the factor p^2, and if we combine its average value with the refraction, R, we shall find that the result agrees very closely with the combined effect of refraction and the sun's parallax, given in Table 1_B, under the heading R'.

Eq. 47 may be used to compute the true azimuth of Polaris and the difference between its apparent altitude and that of the pole, $h' - \phi$. For this purpose we need the quantities a_0, b_0, F, f, and of these a_0 and b_0 are to be obtained from Table 4 with the star's hour angle, t, as the argument. The value of t at any moment is readily found as follows: Let us put

M = Local mean solar time at any moment
θ = Local sidereal time at the same moment
Q = Right ascension of the mean sun at this moment
α = Right ascension of Polaris,

and find from Eqs. 15 and 22,

$$\theta = \alpha + t = M + Q$$
$$t = M + (Q - \alpha).$$

The term $Q - \alpha$, year after year runs through nearly the same succession of values, which are given in Table 3

under the heading D. The small variation of these quantities in different years is taken into account by the numbers Y of Table 2, and replacing $Q-\alpha$ by its value as given in the tables we find

$$t=M+Y+D.$$

The number Y may be assumed constant throughout a year, while D is to be interpolated with the Greenwich mean time, to the nearest tenth of a day, as argument.

Values of the factors f, F are given in Tables 1 and 2 with the argument, the altitude of Polaris, and the year respectively. The value of F may usually be assumed constant for a year and taken from Table 2 without interpolation; in all strictness, however, F varies slightly (aberration + precession) and Table 3 gives, under the heading ΔF, the correction required to reduce the value taken, without interpolation, from Table 2 to its true value at any time. Usually these corrections to F may be ignored, and Tables 1 and 2 are then most conveniently used for the construction of a supplementary table like the following, which will for the given calendar year save all further reference to Tables 1 and 2

1908.	(May).	h	Ff
Y	13^m	$42°$	1.037
			18
F	1.010	43	1.055
			17
R'	1′	44	1.072

Since h can never differ from the latitude by much more than 1°, the above table furnishes all values of the

product Ff that can occur in latitude 43° in the year 1908, and this product when required should be obtained by interpolation from such a table rather than by multiplication of the separate factors, F, f. Construct such a table for your own latitude and write it in pencil at the bottom of Table 3. The formulæ requisite for the use of the tables may now be collected and put in the form shown at the bottom of Table 4.

From these equations it appears that the tables furnish all the data necessary for a determination of azimuth or latitude from observations of Polaris, provided the local mean solar time, M, is known. But all too often the relation of the observer's watch or chronometer to local time is very ill determined, and Table 5 furnishes a method for supplying this defect of data. Its several columns contain the name, magnitude, right ascension and declination of a number of fast moving stars, which may be observed for the determination of time. For the explanation of most of these quantities see the Introduction to the Tables, § 87; but we may here note that the last column of Table 5_A shows under the heading M_0 the mean solar time of transit of each star on the date placed opposite it, and its time of transit on any other date may be found by applying to M_0 as a correction, the gain of sidereal upon mean solar time during the interval between the given date and the date shown in Table 5. Take the amount of this gain from the column PP for D, of Table 3, and add it when the given date is earlier, subtract when it is later than the date given in Table 5.

The values of M_0 increase slowly with the lapse of time, the average rate of increase being a minute in fifty years, and as the tabular quantities are given for the epoch 1925 they may be assumed sufficiently accurate during the first half of the twentieth century. The actual time of a star's transit, M_2, is subject, however, to considerable changes, depending upon the observer's longitude and upon the influence of the leap year cycle of four years. The combined effect of these influences is represented in Table 5*c* from which may be taken, without interpolation, the required correction to adapt M_0 to the given year and longitude. Use as arguments the observer's longitude from Greenwich, and the numbers I, II, etc., indicating that the year for which M_0 is required is the first or second, etc., following a leap year. The presence of an intercalated day, February 29, in leap year requires two tables for such a year, one of which is to be used for the months of January and February, the other during the remainder of the year. See the example § 33.

The use of these tables is as follows: After the instrument has been oriented by means of an assumed value of the watch correction, ΔT, let it be directed to the meridian (south) and the telescope set to an altitude

$$h=(90^\circ-\phi)+\delta,$$

computed for that one of the stars of Table 5 that is next to cross the meridian. Observe by the watch the time, T, at which the star crosses the vertical thread of

the instrument and note whether the resulting correction of the watch

$$\Delta T = M_2 - T$$

agrees with that assumed in computing the orientation. If the difference between the two values of ΔT does not exceed 2^m it may usually be ignored and the orientation and latitude result assumed to be correct. See the following example for a case in which the difference of the ΔTs is too great to be neglected.

33. Illustration of the Use of the Tables.—An observer who supposes his latitude to be about 43° and who possesses a watch approximately set to Central Standard Time is required to make a rough determination of azimuth and latitude at 10 P.M. on May 19, 1908. The relation of the watch to local time is quite unknown, save that it is probably a few minutes slow, and assuming $\Delta T = +10^m$ there is made the following preliminary computation for Polaris and Star No. 29 of Table 5_A, using the auxiliary quantities given on p. 72, which were written in pencil at the foot of Table 3. The subscripts 1, 2 denote quantities that refer to Polaris and Star No. 29 respectively.

Preliminary Computation.

$T+\Delta T$	$10^h\ 10^m$
D	2 12
Y	13
t	12 35
Fb_0	−70′
ϕ	43 0
h_1	41 50
a_0	+14
Ff	1.04
A_1	180 14
* No. 29	June 21
M_0+Table 5C	8 2
33 Days' Change	2 10
M_2	10 12
h_2	20 46

Final Computation.

Obsd. T_2	$10^h\ 9^m$
ΔT	+3
T_1	10 0
$D+Y$	2 25
t	12 28
Obsd. h_1	41° 55′
Fb_0	−1 11
R'	+1
ϕ	43 5
a_0	+11
Ff	1.041
A_1	180 11
Light-Polaris	7 46
Az. of Light	187 57

RECORD OF THE OBSERVATION.

Tuesday, May 19, 1908. At Station Z.

Object.	Circle.	Watch.	Az. Circle.	Vert. Circle.
		h. m. s.	° ′	° ′
Polaris................	L.	10 0 —	180 14	41 55
Light................	L.	....	188 0	0 2
* No. 29	L.	10 9 21	0 0	20 43

A few minutes before 10 o'clock by the watch, Polaris was found, using the computed h_1 to facilitate the finding, the horizontal circle was set to read the computed azimuth, A_1, and at $10^h\ 0^m$ by the watch the star was brought behind the intersection of the cross threads by means of the lower motion, without altering the reading, A_1. The time and circle readings were recorded in the first line of the record as given above. The reading shown in the second line was then taken to a distant light whose azimuth was to be determined, and this reading is the true azimuth of the light provided the assumed ΔT is correct. To guard against error in this assumption the star π Hydræ, No. 29, of Table 5, was observed as set forth at the end of the preceding section.

The difference between the observed time and the computed M_2 gives $+3^m$ as the correction of the watch relative to local mean solar time, and since the difference between this value and the assumed $\Delta T = +10^m$, is too great to be neglected, the observations were recomputed as in the second column, given above under the heading, Final Computation.

The latitude and azimuth there obtained do not exhaust the data furnished by the observations, and we may readily find a more accurate value of ΔT than that above derived. Thus finding the right ascension of π Hydræ from Table 5 and taking Q from the almanac, or computing it by § 19, we obtain the following:

	h	m	s
α	14	1	9
Q	3	49	40
M_2	10	11	29
T_2	10	9	33
ΔT		+1	56

The observed time of transit, T_2, has here been increased by 12 seconds, since the final reduction shows that the reading 0° 0′ at which the instrument was set for the observation of π Hydræ does not correspond to the true meridian, but is 3 minutes east of it. The average southern star requires about 4 seconds of time to move through 1′ of azimuth, and the corrected time is what might have been observed had the instrument been correctly placed.

33a. Artificial Illumination.—For the observation of stars by night there must be provided some artificial illumination for the telescope as well as for the verniers, since, otherwise, the threads that determine the line of sight (cross-wires) will be invisible. For this purpose there are several mechanical devices by which the light from a bull's-eye lantern or electric hand-lamp may be reflected into the field of view of the telescope, but for a small instrument none of these possess any marked

advantage over a bit of candle-grease dropped in the liquid state and allowed to cool upon the center of the objective. Pare it down thin with a penknife and throw the light upon it along a line but little inclined to the axis of the telescope. The effect of the grease upon the optical performance of the objective is quite insensible.

CHAPTER VI.

APPROXIMATE DETERMINATIONS.

34. **Latitude by Circum-meridian Altitudes.** — An obvious method of refining upon the rough determination of a latitude from a single observation of the meridian altitude of the sun or a star (as in § 29) is to measure a series of altitudes during the few minutes preceding and following the maximum h, and to derive from all these observations, which are called circum-meridian altitudes, a better value of the meridian altitude than a single measurement can be expected to furnish. Each measured altitude will usually differ from the maximum altitude by an amount called the *reduction to the meridian*, and this reduction may be accurately computed if either the hour angle or the azimuth of the star at the time of observation is known.

If the observations are made with a sextant, the hour angle will be most convenient for the reduction, and the time of each observation should therefore be noted, to the nearest second, by the use of some watch or other timepiece. To obtain a convenient method of reduction for the observations we put $t = 0$ in the equation,

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t, \tag{48}$$

and obtain for the maximum altitude

$$\sin h_0 = \sin \phi \sin \delta + \cos \phi \cos \delta. \qquad (49)$$

Since in the cases here considered the hour angles are not to exceed 10^m or 15^m, we may put $\cos t = 1 - \frac{1}{2}t^2$, and subtracting the first of the preceding equations from the second obtain,

$$2 \sin \tfrac{1}{2}(h_0 - h) \cos \tfrac{1}{2}(h_0 + h) = \tfrac{1}{2} \cos \phi \cos \delta \,.\, t^2, \qquad (50)$$

which is approximately equivalent to,

$$h_0 - h = \frac{\cos \phi \cos \delta}{2 \cos h_0} \cdot \frac{t^2}{\sin 1''}. \qquad (51)$$

This is the equation of a parabola having h_0 for its maximum ordinate and h and t for rectangular coordinates; and we may infer from it that if the sextant readings be plotted as ordinates upon cross-section paper with the observed times as abscissas, the resulting curve will be a parabola whose maximum ordinate will be the sextant reading corresponding to the maximum altitude of the body observed.

This maximum ordinate may be read directly from the curve, or it may be derived with greater precision by means of the theorem that the area included between a parabola and any chord perpendicular to its axis, equals two thirds of the length of this chord multiplied by its distance from the vertex, $A = \frac{2}{3}xy$. The intercept of the plotted curve upon the axis of x, or upon any line parallel to this axis, is such a chord, whose

length may be directly measured, and the distance of the vertex from this axis is the quantity sought. If, therefore, the length of the intercept, x, and the area of the corresponding part of the curve, A, be directly measured, we have at once,

$$y_0 = \frac{3}{2}\frac{A}{x}. \tag{52}$$

Friday May, 4 1897.

Sextant No. 5096. Index corr. $-3' \, 34''$. Observer, C.

Barometer 29.10. Thermometer 69° Fahr.

Horizon Roof Direct.

Limb.	Watch.	Sextant.
	h. m. s.	° ′ ″
L.	11 44 33	125 36 10
L.	46 15	125 39 0
U.	50 27	126 47 45
U.	51 23	126 48 20
L.	53 15	125 46 10

Horizon Roof Reversed.

Limb.	Watch.	Sextant.
	h. m. s.	° ′ ″
U.	11 55 45	126 50 5
U.	58 3	126 49 45
U.	12 0 8	126 48 0
L.	5 43	125 34 40
L.	6 30	125 32 30

The author finds from the area of the curve the following value of y_0, from which the latitude is derived as below.

y_0	126° 18′ 36″
Index corr.	−3 34
h'	63 7 31
Ref.-Par.	−0 24

h_0	63° 7′ 7″
z_0	26 52 53
δ	16 11 41
ϕ	43 4 34

The axis of the plotted parabola intersects the time scale at 11^h 55.0^m, and comparing this number with the local mean time of apparent noon, $12^h + E = 11^h\ 56^m.6$, we obtain as the correction required to reduce the watch to local mean solar time $\Delta T = +1.6^m$. A value of ΔT thus determined may easily be in error by 20 or 30 seconds.

Among the advantages of this mode of treatment of the data may be noted that each observation contributes its appropriate share toward determining the maximum altitude of the body, and that no knowledge of the error of the timepiece is required. In fact the correction ΔT

may be approximately determined from the curve, by noting, as the chronometer time of apparent noon, the point at which the axis of the parabola intersects the axis of x.

Let the student plot the preceding observations made upon the sun's upper and lower limb, and derive from the area of the curve the sextant reading corresponding to the sun's meridian altitude. Before plotting, each sextant reading, double altitude, must be corrected by twice the sun's semi-diameter, interpolated from the almanac for the date of observation, i.e., $\pm 31' 47''$, in order to obtain the corresponding reading to the sun's center.

For an approximate method of determining latitude from altitudes of Polaris the student may consult the American Ephemeris, Table IV, and explanations at the end of the volume.

35. **Reduction to the Meridian.** — If circum-meridian altitudes are to be measured with a theodolite, it will usually be convenient to orient the instrument and determine from a reading of the horizontal circle the azimuth corresponding to each observation. A graphical solution may then be made precisely as in the case of the observed times treated in the preceding section, or we may derive from Equations 15,

$$\sin \delta = \sin \phi \sin h - \cos \phi \cos h \cos A, \qquad (53)$$

and from this, by the method of § 34, we find the relation,

$$h_0 - h = \cos \phi \cos h_0 \sec \delta \cdot \frac{A^2}{2} + \text{etc.} \qquad (54)$$

Through this equation and the known values of A, compute for each observed altitude its own reduction to the meridian.

The quantities $h_0 - h$ and A are here supposed to be expressed in radians, but in practice it is convenient to express the azimuth in minutes and the reduction to the meridian in seconds of arc. Representing the azimuth when so expressed by a', we make in Equation 54 the following substitutions:

$$A \text{ (radians)} = a' \cdot \frac{60}{206265}, \quad (h_0 - h) \text{ (radians)} = \frac{(h_0 - h)''}{206265},$$

and uniting into one, all the numerical factors that are found in the equation as thus altered, and introducing the symbol f as an abbreviation for the product of all factors not containing a', we obtain,

$$\begin{aligned} f &= [7.9407] \cos \phi \cos h_0 \sec \delta, \\ (h_0 - h)'' &= f(a')^2. \end{aligned} \qquad (55)$$

The accents, $'$, $''$, denote that the marked terms are expressed in minutes and seconds respectively. Use an estimated, approximate, value of h_0 for the computation of f.

The preceding results cannot be directly applied to a star north of the zenith, since for such a star the azimuth, A, is a large quantity; but if the azimuth be reckoned from the north point instead of from the south, i.e., if we put $a_0 = 180° - A$, we may derive formulæ identical with the above, which therefore apply to this case when a_0 is defined as the supplement of the azimuth. For a

star at lower culmination, i.e., on the meridian below the pole, the altitude is a minimum instead of a maximum, and the reduction to the meridian must therefore be given the negative sign. Note that this can be accomplished in Equation 55 by considering δ to represent the supplement of the star's declination instead of the declination itself. These formulæ for reduction to the meridian should not be applied in the case of stars whose hour angles exceed 10^m or 15^m. For an application of the formulæ see § 73.

36. Time from Altitudes near the Prime Vertical.—With a sextant an approximate determination of time is best made by measuring a series of altitudes of the sun or a star when the body is, as near as may be, due east or west, noting the chronometer time, T, of each observation.

The formulæ for the transformation of coordinates furnish for each such observation the equation,

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t,$$

which is readily transformed into,

$$\cos t = \sec \phi \sec \delta \sin h - \tan \phi \tan \delta, \tag{56}$$

and by means of this equation the hour angle corresponding to each observed time may be derived. The chronometer correction will then be furnished by one of the following equations:

$$\begin{aligned} &\text{For the Sun, } \Delta T = E + t - T, \text{ Local Mean Solar Time.} \\ &\text{For a Star, } \Delta T = a + t - T, \text{ Local Sidereal Time.} \end{aligned} \tag{57}$$

The symbol E denotes the equation of time. Its numerical value is most conveniently derived from the Solar Ephemeris, p. 400 of the almanac.

DOUBLE ALTITUDES OF ARCTURUS, NEAR EASTERN PRIME VERTICAL.

Wednesday, March 29, 1899.

Sextant, Cameron. Chronometer, B. Observer, C.
Index Corr. $+18' \, 37''$. Barom. 28.81. Therm. $+19°$ Fahr.

Sextant.	Chronometer.	$a+t$.	ΔT.
° ′	h. m. s.	h. m. s.	s.
54 30	9 30 5.5	9 29 5.6	−59.9
55 0	31 29	30 27.9	61.1
55 30	32 50	31 50.2	59.8
	Horizon roof reversed.		
56 0	34 14	33 12.5	61.5
56 30	35 36	34 34.8	61.2
57 0	36 58.5	35 57.1	61.4
		Mean ΔT..........	−60.8

	° ′ ″		° ′ ″	° ′ ″
ϕ	43 4 37	Corr'd Sext.	54 48 37	57 18 37
δ	19 42 9	App't h	27 24 18	28 39 19
sec ϕ	0.13642	Refraction	1 55	1 49
sec δ	0.02620	h	27 22 23	28 37 30
tan ϕ	9.97082	sin h	9.66255	9.68040
tan δ	9.55401	sec ϕ sec δ sin h	9.82517	9.84302
sec ϕ sec δ	0.16262	tan ϕ tan δ	9.52483	9.52483
		Subtract	9.99862	0.03367
	h. m. s.	cos t	9.52345	9.55850
a	14 11 6.1	$-t$	4 42 0.5	4 35 9.0
		$a+t$	9 29 5.6	9 35 57.1

Adopted $\Delta T = -60^s.8$ (Local sidereal).

In place of the laborious process of separately reducing each observed altitude, we may usually treat the mean of the sextant readings and the mean of the observed times as if they constituted a single observation. When the observed body is near the prime vertical the time interval covered by a set of observations which it is purposed to unite into a mean result may extend to 15 or 20 minutes without sensible error, but the error

of the process increases rapidly with increasing distance from the prime vertical, and the time interval must be correspondingly diminished.

In the preceding example of a time determination from sextant altitudes, the sextant was set accurately to a set of readings differing by a uniform interval of 30′, and the times noted at which the observed body came to the corresponding altitudes. In the reduction advantage is taken of this circumstance by computing the value of $a+t$ for the first and last observations only, and interpolating the intermediate values. Observe that the columns $a+t$ and ΔT, although placed near the beginning of the reduction, are really the last to be filled out.

37. Azimuth Observations at Elongation. — An excellent approximate determination of the azimuth of a terrestrial mark may be made by measuring, with a theodolite, the horizontal angle between the mark and a circumpolar star at the time of its elongation, i.e., its maximum digression from the meridian.

It may be seen by inspection that at the instant of elongation the astronomical triangle, Fig. 3, is right-angled at the star, and we obtain from it,

$$\begin{aligned} \sin A_e &= \cos \delta \sec \phi, \\ \cos t_e &= \cot \delta \tan \phi, \end{aligned} \tag{58}$$

where the subscript e shows that the quantities thus marked relate to elongation only, and A_e is measured from the north toward either east or west as the case may require.

The time of elongation is then given by

$$\left.\begin{array}{ll}\text{Sidereal.} & \theta_e=\alpha \pm t_e \\ \text{Mean solar.} & M_e=\alpha - Q \pm t_e\end{array}\right. \quad \left.\begin{array}{l}+W. \\ -E.\end{array}\right\} \text{Elongation} \qquad (59)$$

If H denote the measured angle between the star and the mark, positive when the mark is east of the star, we shall have as the azimuth of the mark

$$A_m = 180° + A_e + H. \qquad (60)$$

Equations 58, 59, 60 leave nothing to be desired on the score of simplicity and the only observation required in connection with them is the measurement of the angle H at the time θ_e or M_e. This is frequently done by pointing the instrument upon a star a little before the computed time, M_e, and following the star's motion in azimuth until it ceases to move away from the meridian. The verniers are then read and immediately thereafter a single pointing upon the azimuth mark is made. This method is convenient when the time is very imperfectly known but, in general, a better procedure is to determine the time to the nearest minute, by the method of § 33, or otherwise, and to make at least two pointings upon both mark and star, one Circle R and one Circle L, to eliminate instrumental errors, since this elimination is of far more consequence than any precise agreement between the time of elongation and the actual time of observation.

Indeed the error introduced by failure to measure the angle when the star is exactly at elongation, may be

completely remedied as follows: Since the star's azimuth is a maximum at elongation, its azimuth τ minutes before or after elongation, which we represent by A_τ. must be less than A_e by some small quantity depending upon τ^2. Eq. 58 shows that we may diminish A_e to any desired amount by increasing δ and we therefore put

$$\sin A_\tau = \cos(\delta + x) \sec \phi, \tag{61}$$

and seek to determine the value of x. By direct computation from Eq. 14, or by development of this equation by Taylor's Formula, we may find the values of x corresponding to $\tau = 10^m$, which are tabulated below with the declination as argument, and in terms of these quantities we shall have for any value of τ not exceeding 20 minutes

$$x = x_{10}\frac{\tau^2}{100} \tag{62}$$

Introducing this value into Eq. 61 we may readily compute the azimuth of any close circumpolar star that is within twenty minutes of its elongation. For the reduction of an observation consisting of several pointings upon such a star, e.g., a set of "repetitions," substitute for τ^2 in Eq. 62 the mean of the (τ minutes)^2 corresponding to the several pointings.

ELONGATION CORRECTIONS TO δ FOR $\tau = 10^m$.

δ	x_{10}
85°	+17″.1
86	13 .7
87	10 .3
88	6 .9
89	3 .4
90	0 .0

The following illustrative observations at elongation were made with an engineer's transit, using the method of repetitions (§ 53), two pointings in the set, with a reversal of the instrument between them. After reversal the bubble of the azimuth level was brought back to the initial position, as is shown by the level readings, and the instrumental errors may be considered as well eliminated.

Wednesday, May 14, 1902.

At Station A. Inst. No. 306. Obsr. C.

Object.	Circle.	Pointing.	Watch.	Horizontal Circle.		Levels.	
				Ver. A.	Ver. B.	W.	E.
δ Urs. Min...	L.	1	$8^h\ 44^m$	$4°\ 39'\ 5''$	$39'\ 5''$	9.8	12.2
Az. mark...	R.	2	8 49	251 24 0	24 0	12.3	10.0
						+0.15	

The measured angle between star and mark is

$$H = \tfrac{1}{2}(251°\ 24'\ 0'' - 4°\ 39'\ 5'') = 123°\ 22'\ 27''.5,$$

and the further reduction of the observation is as follows:

ϕ	$43°\ 4'\ 37''.0$	x_{10}	$+11''.7$
δ	86 36 44 .8	x	+ 0 .8
$\cot\delta$	8.7723	$\delta+x$	$86°\ 36'\ 45''.6$
$\tan\phi$	9.9708	$\cos(\delta+x)$	8.77148
t_e	$5^h\ 47^m\ 18^s$	$\sec\phi$	0.13642
α	18 4 1	A_T	184 38 23 .2
Q	3 27 36	H	123 22 27 .5
M_e	8 49 7	A_M	308 0 51

The recorded times of observations are from a watch approximately regulated to Central Standard Time and supposed to be about two minutes slower than local

time. Allowing for this error the values of τ for the two pointings are -2^m and $+3^m$ respectively, and we have therefore $x = x_{10} \cdot \frac{4+9}{2 \times 100}$ from which to determine x. On account of the small values of τ, tenths of seconds are here retained in the computation in order to illustrate the method, but the result for A_M can pretend to no such degree of accuracy.

A frequent source of substantial error in similar determinations lies in the assumed latitude, and unless this is well known the elongation method should be either avoided altogether or supplemented by observing the elongation of another star on the opposite side of the pole. In the above case an error of $1'$ in the assumed ϕ would vitiate A_M to the extent of $4''.5$, but this error could be in great part eliminated by observing also the western elongation of 51 *H.* Cephei which occurs about half an hour after that of δ Urs. Min. Adopt the mean of the resulting values of A_M. It is always advantageous to make the mean of the pointings coincide as nearly, as may be with the computed time of elongation in order to eliminate any effect arising from error of the time piece employed.

38. Time and Azimuth from Two Stars.—The methods of § 32 may be refined so as to furnish the required azimuth, latitude or time with any desired degree of accuracy, but such refinement naturally is at the cost of increased complexity and increased labor in their application, e.g., in the observations we may so use the instrument, Circle R and Circle L, as to eliminate its

PLATE II.

An American Theodolite. Diameter of Horizontal Circle 8 inches.
Approximate Cost $400.

[*To face p.* 90.]

errors of collimation, level, etc.; we may resort to the method of repetitions, § 53, to secure increased accuracy in the horizontal angle between mark and star, etc. Corresponding to the increased precision thus obtained there is required a more rigorous method of reduction for the observed data and the mathematical theory of such a method is as follows:

Let M denote a terrestrial mark between which and some known star there has been measured the horizontal angle, H, corresponding to the time, T, as indicated by some watch or other time piece, and let h denote the altitude of this star measured simultaneously with H. See the following example for an illustration of the method of observing:

Let us put

$$\Delta T = U + x = \text{The Chronometer Correction (Sidereal)}$$
$$A_M = A_0 + y = \text{True azimuth of the mark}$$

where U and A_0 are provisional values of ΔT and A_M obtained from the orientation method, § 32, or otherwise, and x, y, are the unknown corrections required to transform U and A_0 into ΔT and A_M. To determine the values of x and y we note that the true azimuth of any star observed may be expressed in the form,

$$A = A_0 \pm H + y = A_1 + y, \qquad (63)$$

where A_1 is employed as an abbreviation for the measured approximate azimuth, $A_0 \pm H$. Through the first of Eq. 12 we may also compute the star's azimuth A

from the observed time T, and the known h, δ and α, thus

$$t = T + (U + x) - \alpha = \tau + x$$

$$\sin A = \cos \delta \sec h \sin (\tau + x) \qquad (64)$$

where $\tau = T + U - \alpha$

is introduced as an abbreviation for the second member of the last equation. Let us further put as abbreviations:

$$\sin A' = \cos \delta \sec h \sin \tau$$

$$C' = \cos \delta \sec h \cos \tau \qquad (65)$$

and expanding $\sin (\tau + x)$ in Eq. 64 find

$$\sin A = \sin A' \cos x + C' \sin x. \qquad (66)$$

When x and $\sin A'$ are small quantities whose cubes may be neglected, we may put $\cos x = 1$, without sensible error, and find by transposition and development of $\sin A - \sin A'$,

$$2 \sin \tfrac{1}{2}(A - A') \cos \tfrac{1}{2}(A + A') = C' \sin x. \qquad (67)$$

Again neglecting terms of the order x^3 this expression becomes

$$A = A' + C' \sec A' \cdot x, \qquad (68)$$

which when subtracted from Eq. 63 furnishes the relation

$$y - C' \sec A' \cdot x = A' - A_1. \qquad (69)$$

The quantities y and $A' - A_1$ of this equation should be expressed in seconds of arc, but the chronometer correction, x, will be required in seconds of time, and combining with sec A' the numerical factor 15 required for this purpose, we call the product G and put the coefficient of x in the form $C'G$. Values of log G are given in the following short table.

THE G FACTOR.

A'	log G	A'
$\pm 0°$	$-1.1761+$	$180° \pm 0°$
1	.1762	1
2	.1764	2
3	.1767	3
4	$-1.1772+$	4

The symbols $+$, $-$ placed adjacent to log G indicate the essential sign of G, $+$ for a northern, and $-$ for a southern star, and they must be heeded in computing the coefficient of x.

Every observed star will furnish an equation of the type (Eq. 69), and two stars will therefore suffice for the determination of both y and x. It is a matter of consequence, however, to choose wisely the particular stars that are to be observed. One of them should always be near the pole and Polaris should usually be chosen wherever it may be in its diurnal path. The other star should be on the opposite side of the zenith from Polaris, and because of the assumptions made in

Eq. 66 it should be observed only when near the meridian; e.g., when x is as great as 3^m the star's azimuth should not exceed $\pm 3°$.

We now collect and arrange our formulæ as follows:

$$A_1 = A_0 \pm H$$

$$\tau = T + U - \alpha$$

$$C = G \sec h \cos \delta \cos \tau$$

$$\sin A' = \sec h \cos \delta \sin \tau$$

$$y + Cx = A' - A_1 \quad (2 \text{ Equations})$$

$$A_M = A_0 + y$$

$$\Delta T = U + x.$$

Example.—The application of the method is illustrated by the following record and reduction of observations made with the instrument shown in Plate I, and an ordinary watch.

Tuesday, April 23, 1907.
At Station E. Azimuth of Asylum Light.
Observer, C.

Object.	Circle.	Watch.	Horizontal Circle.		Vertical Circle.		Levels.
			Ver. A	Ver. B	Ver. I.	Ver. II.	
		h m s	° ′ ″	′ ″	° ′ ″	′ ″	Azimuth Level
Light.....	R.	9 0 —	187 54 0	53 45	....	....	W. E.
δ Crateris..	R.	2 16.5	358 14 15	14 15	32 40 45	39 15	25.7 9.3
Polaris....	R.	5 11	179 5 40	5 30	42 9 30	7 0	9.1 25.6
Polaris....	L.	10 14	359 8 30	8 25	42 3 15	3 45	
δ Crateris..	L.	14 8.5	181 41 20	40 45	32 38 45	39 0	+0.15
Light.....	L.	9 17 —	7 54 30	54 10	....	....	Altitude Level
							N. S.
							2.0 8.0
							8.0 2.2
							−0.1

The instrument was approximately oriented before beginning the observations, a precaution that it is always well to take, since we may then assume as the provisional azimuth of the mark, A_0, the mean of the readings of the horizontal circle corresponding to it. Similarly, taking the mean of the observed times for each star and comparing the time for δ Crateris with the right ascension of that star, we find with sufficient approximation, $U = +2^h\ 6^m$. The instrument was relevelled when reversed by bringing the bubble back to to the position it occupied at $9^h\ 5^m$ and the bubble readings in the last column show that in the mean the errors of level are so small that their effect may be ignored. See §§ 42 and 50. The further reduction of the observations may be made as follows:

Star.	Polaris	δ Crateris
δ	88° 48′ 31″	−14° 16′ 41″
α	$1^h\ 24^m\ 46^s$	$11^h\ 14^m\ 42^s.3$
T	9 7 42	9 8 12 .5
$T+U$	11 13 42	11 14 12 .5
τ	9 48 56	−0 0 29 .8
τ (arc)	147° 14′ 0″	−0° 7′ 27″
h	42 6′	32 39
G	1.1762	1.1761n
cos τ	9.9247n	0.0000
sec h	0.1296	0.0747
cos δ	8.3179	9.9864
sin τ	9.7334	7.3359n
C	9.5484n	1.2372n
sin A'	8.1809	7.3970n
A'	179° 7′ 52″	359° 51′ 25″
H	8 47 5	172 3 33
A_1	179 7 1	359 57 39

$y - 0.35x = +51''$
$y - 17.27x = -374$

A_0	187 54 6
y	+60
A_M	187 55 6

U	$+2^h\ 6^m\ 0^s$
x	+25.1
ΔT	+2 6 25.1
Q	2 4 0.3
ΔT	+ 2 24.8

The two values of ΔT above derived relate respectively to sidereal and to mean solar time. A com-

parison of the watch with the standard clock of the Washburn Observatory made immediately after this observation gave as its true correction, $\Delta T = +2^m 24^s.6$.

In the reduction, the coordinates of the stars and the quantity Q have been taken from the almanac. For comparison let the student derive their values from the tables at the end of this book.

The altitudes, h, that appear in these formulæ need not be observed with great precision, an error of $1'$ or even $2'$ having in general no appreciable effect upon the values of x and y. Indeed the observations of altitude may be entirely omitted and adequate values of h computed from the orientation tables for Polaris, and from the declinations and hour angles for the southern star. If, however, these altitudes are accurately observed their values, corrected for refraction and instrumental error, may be made to yield an excellent determination of the latitude. We may use for their reduction, in the case of the southern star, the methods of §§ 34, 35 and for Polaris the following equivalent of Eq. 44.

$$\phi = h - p \cos(\tau + x) + C''$$

$$C'' = (2(180 - A_1{}^\circ))^2 \sin 2h.$$

In the last equation the value of A_1 must be expressed in degrees and the resulting value of C'' will then be given in seconds of arc, e.g., if $A_1 = 178^\circ\ 50'$ $h = 45^\circ$, we find

$$C'' = \overline{2 \times 1 \cdot 2}^2 \sin 90^\circ = 5''.8.$$

It may be noted that when the chronometer correction, $\varDelta T$, is sufficiently well known we may put $x=0$, omit the observation of the southern star and find A_M from the equation furnished by the Polaris observation alone. Similarly, if the azimuth of the mark is given, we may put $y=0$, omit the observation of Polaris and determine $\varDelta T$ from an observation of a southern star, or, by daylight, from an observation of the sun when near the meridian. Compare with the method of § 31. When applied to a star this procedure will often be of advantage in connection with precise determinations of azimuth.

Errors of Levelling.—A major source of error in results obtained as above is found in imperfect levelling of the instrument, and it should be avoided by reading the azimuth level (§ 50) at some time during the first half of the set of observations and relevelling the instrument after reversal. Bring the bubble back to the exact place that it occupied in the tube when first read and record the second readings. Whatever error affected the first half of the set will now be balanced by an equal and opposite error in the second half.

It may happen that the recorded level readings, before and after reversal, do not agree, and in this case the angle, b', that the vertical axis makes with the plane of the meridian may be found from the recorded level readings (§§ 42, 43) and a corresponding correction applied to the time and azimuth found through the vitiated values of x and y. From the spherical triangle

formed by the pole, the zenith and the point of the sky towards which the vertical axis is directed we find,

$$\Delta x = +b' \sec \phi \qquad \Delta y = +b' \tan \phi,$$

which are the required corrections for level error. Count b' as positive when, on the whole, the east end of the azimuth level is higher than the west end. Assuming the value of a level division to be $15''$ ($d = 7''.5$) we find from the observations on p. 94, $b' = +1''$, $\Delta x = +1''.3 = +0^s.1$, $\Delta y = +1''$, which are quantities quite inappreciable for the present purpose.

CHAPTER VII.

INSTRUMENTS.

In the several determinations thus far considered we have for the most part assumed that the data furnished by the instruments employed were free from purely instrumental errors, and in approximate work this may usually be done if due care has been bestowed upon the adjustments. But where a higher degree of precision is required it becomes necessary to study the instrument employed, as being in itself a source of errors that need to be eliminated, and we must turn therefore to a more detailed consideration of some of the instruments used in field astronomy before taking up the class of methods called accurate.

42. The Spirit-level. — The spirit-level is used in astronomical practice to measure small deviations of a line or surface from a vertical or horizontal position, and incidentally to adjust a part of an instrument to such a position. It consists essentially of a glass tube bent or ground into an arc of a circle of large radius and so mounted that the plane of this circle is approximately vertical. The tube being nearly filled with ether and its ends hermetically sealed, the small volume of air or

vapor that remains in the tube is collected into a bubble which always stands at the highest point of the circle, so that a line drawn from its middle point through the centre of curvature of the tube is vertical. The upper surface of the tube is usually provided with a scale of equal parts, and the position of the bubble in the tube is determined by the *readings* of its ends upon this scale. The angle subtended at the centre of curvature of the tube by the space between two consecutive lines is called the *value of a division* of the level, and this value, which will be represented by $2d$, is required for transforming the indications of the level into seconds of arc. Note that d represents one half the value of a level division.

Let such a level be supposed attached to a theodolite, the inclination of whose vertical axis to the true vertical is to be determined from readings of the bubble. We are here concerned with angular measurements, e.g., the angle that the axis makes with the true vertical; the angle moved over by the level bubble, as seen from the centre of curvature of the tube, when the instrument is turned from one position to another; etc., and as the simplest method of dealing with these angles we shall imagine the whole apparatus projected radially upon the celestial sphere, so that the arc joining the points in which any two projected lines meet the sphere, measures the angle between these lines. This method of analysis by projecting the parts of an instrument upon the sphere is in common use, and the student should acquire a clear conception of the simple case to which it is here first applied.

To determine the relation of the bubble readings to the required inclination, we imagine the axis of the instrument and the plane of the level extended until they meet the celestial sphere, as in Fig. 7, which represents

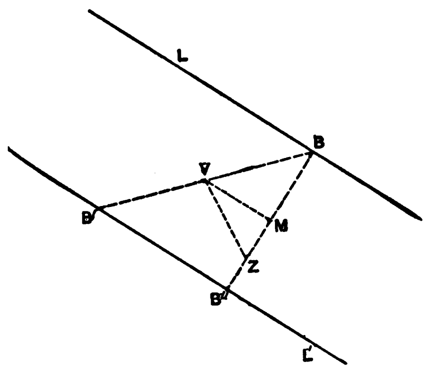


Fig. 7.—Theory of the Spirit-level.

a small part of the sphere adjacent to the zenith, Z. In this figure V is the point in which the produced axis meets the sphere, and LB is the trace of the plane of the level tube upon the sphere. The projection of the middle of the bubble upon the sphere must be at B, the point in LB nearest to Z, and found, therefore, by letting fall a perpendicular from Z upon LB. If, now, the theodolite be turned 180° in azimuth, i.e., rotated about V as a pivot, the level tube will be revolved about V as a centre, into the position $L'B'$, and the point B will fall at B', but the middle of the bubble will stand at B'' instead of B', since this is now the point nearest to the zenith. From elementary geometrical considerations, $VM = \frac{1}{2}B'B''$, where $B'B''$ is the space moved over by the level bubble when the instrument is turned from

one position to the other, and VM is the projection upon the plane of the level tube of the arc, VZ, that measures the angle between the axis of rotation and the true vertical. Calling this projection b and representing by a', b', a'', b'' the scale readings of the ends of the bubble in the two positions, we have

$$b = \tfrac{1}{2}\left(\frac{a'+b'}{2} - \frac{a''+b''}{2}\right)2d = \frac{(a'-b'')+(b'-a'')}{2}d. \quad (79)$$

It is customary to record the several readings in the form,

(Symbols used above.)		(Actual observations.)	
N	S	N	S
a'	b'	16.4	32.2
a''	b''	39.6	9.1
		7.35	

The letters N and S denote the north and south ends of the level tube, or some equivalent system of distinguishing between them.

43. Discussion of the Level Readings. — The student should now note that:

(*a*) The coefficient of d in Equation 79 is the mean of the diagonal differences in the square array formed by the four numbers tabulated in the preceding example. This example represents the manner in which level readings should be recorded, and the mean of the diagonal differences, 7.35, written below the line, should be worked out and entered with the record.

(*b*) If the bubble readings have been correctly taken and there is no change in the length of the bubble during the observation, these differences must be equal, one

to the other, thus furnishing a check upon the accuracy of the level readings, which should always be applied immediately after recording them. If the temperature is changing rapidly, the length of the bubble may be changed and the check impaired without necessarily diminishing the accuracy with which b is determined.

(*c*) If the greatest of the four numbers stands in the column marked N, the north end of the level tube is on the whole higher than the south end, and the vertical axis is tipped toward the south. Determine the sign of b in this manner.

(*d*) The zero of a level graduation is sometimes placed at one end of the scale and sometimes in the middle, but the method of record and reduction given above applies to both cases.

(*e*) It is apparent from the figure that the point of the level tube midway between B' and B'' marks that radius of the level tube which is most nearly parallel with the rotation axis of the instrument. Since this radius ought to pass through the middle point of the scale, and does so pass when the level is in adjustment, we have as the error of adjustment of a level numbered continuously from one end to the other,

$$\epsilon = \frac{s}{2} - \frac{a' + b' + a'' + b''}{4}, \qquad (80)$$

where s represents the total number of divisions in the level scale. In the example given above $s = 50$ and $\epsilon = 0.7$ division.

The essential element in the determination of b is

the *reversal* of the level with the resulting displacement of the bubble, and it is a matter of indifference whether this displacement is produced by revolving the level about a vertical axis to which it is attached, as in the case considered above, or by picking the level up bodily from a plane or line upon which it stands, turning it end for end and replacing it in the reversed position, as is done in measuring the inclination of an approximately horizontal axis. Let the student show that the inclination of this axis to the plane of the horizon may be obtained from the bubble readings exactly as the inclination of the vertical axis was determined above. The greatest of the four readings is adjacent to the high end of the axis. Determine in this way the inclination of the horizontal axis of a theodolite.

A fine level is an exceedingly sensitive instrument and requires great care in its use. Unless unusually well supported its readings may be vitiated by the observer passing from one side of it to the other, or even by shifting his weight from one foot to the other. Therefore observe the following precepts:

1. Keep away from the level as much as possible.

2. Don't allow the sun to shine upon it.

3. Don't hold a source of heat, e.g., a lamp or your own hand, near a level longer than is strictly necessary.

4. If the level has a chamber with reserve supply of air at one end of the tube, use it to regulate the length of the bubble, keeping this always about one half as long as the scale.

5. Make the inclinations that are to be measured

as small as possible, in order to avoid any considerable run of the bubble and the resulting effect of possible irregularities in the level tube.

44. Value of a Level Division.—The value of a level division is most conveniently determined by measuring with a micrometer, or finely graduated circle, the vertical angle through which its tube must be tipped in order to cause the bubble to run past a given number of divisions of the scale. If the necessary apparatus for such a determination is not at hand, the following method will furnish equally good results and requires only an engineer's transit, to which the level must be attached with its plane approximately vertical.

Let the instrument be firmly set up but very much out of level, e.g., with its vertical axis making an angle, γ, with the true vertical amounting to 2°, more or less. See p. 109 for a method of determining the exact value of this angle, which will be required in the reduction of the observations. If the transit is now turned slowly about its vertical axis (azimuth motion), the level-bubble will run back and forth in its tube, and two positions of the instrument may be found at which the bubble will come to the middle of its scale. We shall designate the readings of the azimuth circle corresponding to these two positions by A_1 and A_2.

Any slight turning of the instrument either way from A_1 or A_2 will cause a corresponding slight motion of the bubble, and to determine the relation of the bubble readings to the corresponding circle readings we resort to Fig. 8, which represents a portion of the celestial

sphere adjacent to the zenith, Z. V is the point in which the deflected axis of the instrument meets the sphere, and SV is the trace upon the sphere of the plane of the level-tube, which is assumed to have been adjusted approximately parallel to the vertical axis of the transit. Small errors in this adjustment are of no consequence.

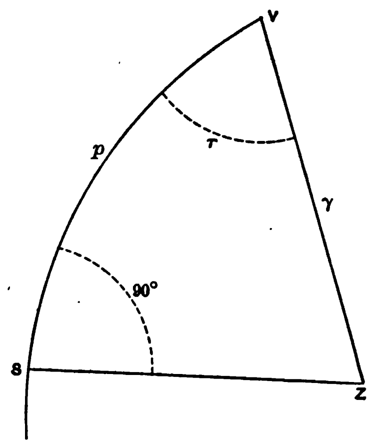


FIG. 8.—Determination of *d*.

As the instrument is turned in azimuth, carrying the level-tube with it, the arc SV must revolve about V as a pivot, and the amount of its rotation will be measured by the successive readings of the azimuth circle. It may be seen readily that the angle τ of the figure corresponding to any particular circle reading, A, is given by the equation

$$\tau = \tfrac{1}{2}(A_1 + A_2) - A, \qquad (82)$$

$\frac{1}{2}(A_1 + A_2)$ being the circle reading at which SV coincides with VZ.

Since a level-bubble always stands at the highest point of its tube, the point nearest the zenith, we may find the point in the figure corresponding to the middle of the level-bubble by drawing from Z an arc of a great circle perpendicular to SV, and the intersection, S, will be the required point. In the right-angled spherical triangle SVZ thus formed we have the relation,

$$\tan p = \tan \gamma \cos \tau, \qquad (83)$$

in which p measures the distance of the middle of the bubble from the fixed point V. To find the effect upon p of any small variation in τ, i.e., to find how far the bubble will run when the instrument is turned slightly in azimuth, we differentiate this equation and obtain

$$-dp = \tan \gamma \cos^2 p \sin \tau \, d\tau, \qquad (84)$$

and substituting in place of these differentials, small finite increments of the respective quantities, we obtain

$$2d(b' - b'') = \tan \gamma \cos^2 p \sin \tau \, (A' - A''), \qquad (85)$$

where d represents the value of half a level division and b' and b'' are the scale readings of the middle of the bubble, corresponding to the circle readings A' and A''.

Equation 85 may be used to determine the value of d, but whenever ordinary care is bestowed upon the adjustment of the level, i.e., to make the radius passing through the middle point of the scale parallel to the vertical axis of the theodolite (Equation 80), the readings A_1 and A_2 will be so nearly $180°$ apart that we may put $\tau = 90°$, $\cos p = 1$, for all positions of the bubble within

the limits of its scale, and thus obtain in place of Equation 85 the simpler relation

$$d=\frac{A'-A''}{2(b'-b'')}\tan \gamma. \tag{86}$$

In this equation $A'-A''$ and $b'-b''$ are to be derived from the readings of the horizontal circle and level, respectively, and in making observations for their determination it is well to bring the bubble as near as may be to one end of the tube and set the circle to read the nearest integral $10'$. Then turn the instrument to each successive $10'$ or $20'$ reading, and record the readings of the bubble until the former has traversed the entire length of its scale, after which repeat the operation in the inverse order, using the same circle settings as before. With reference to the direction of the bubble's motion these two series will be designated as Forward and Backward. Having completed these observations, turn the instrument to the second position in which the bubble plays, e.g., from A_1 to A_2, and make a similar double set of readings.

The readings obtained at any two settings of the instrument will determine values of $A'-A''$ and $b'-b''$, and therefore a value of d, but it is advisable to secure a considerable number of these determinations, ranging over the whole length of the level-tube, in order to test its uniformity. Supposing such a series to have been made, the manner of forming the differences $b'-b''$ illustrated below, may be followed with advantage, i.e., subtract the first b from the first one following the

middle of the set, the second b from the second one after the middle, etc.

The angle γ of Equation 86 should be determined at the time of deflecting the axis, as follows: After having carefully levelled the instrument, take a reading of the vertical circle when the line of sight is directed toward a fixed mark, that we may call P. By means of the levelling screws deflect the axis exactly toward or from P through some convenient angle, e.g., $1°$ if the vertical circle reads to seconds, $3°$ if it reads only to minutes, and again point upon P and read the circle. The difference of the two readings is the value of γ. To make sure that the deflection of the axis is made in the proper direction, by means of the levelling screws make the reading of the azimuth level (see § 50) the same after deflection that it was before deflection, and there will then be no component of deflection perpendicular to the direction P.

45. Example. — We have the following example of the record and reduction of the first half of a complete set of observations for the determination of d. In the reduction we note that the divisor $2(b'-b'')$ of Equation 86 is equivalent to $2b'-2b''$, and since b is the scale reading of the middle of the bubble, $2b$ is equal to the sum of the readings of the ends of the bubble. The column headed $2b$ is found in this way from the mean of the two sets of bubble readings opposite each circle reading.

The regular progression of the numbers in the column $2(b'-b'')$ suggests a level-tube of variable curvature, but the amount of data is not sufficient to decide this with certainty. More observations are needed.

Friday, Dec. 7, 1894.
Alidade Level of Universal Instrument, No. 2598.
Readings to Mark.

	Mic. I.	Mic. II.	Mean.
Axis Vertical.	180° 26′ 45″	26′ 45	180° 26′ 45″
Axis Deflected.	179 27 0	27 6	179 27 3
		$\gamma =$	0 59 42

Azimuth Circle.	Bubble. Forward.		Bubble. Backward.	
291° 0′ 0″	26.3	0.2	26.0	0.4
10 0	28.1	2.3	27.7	1.9
20 0	30.2	4.2	29.7	3.9
30 0	32.3	6.4	32.1	6.2
40 0	34.0	8.1	34.2	8.5
50 0	36.2	10.2	36.5	10.6

$2b$	$2(b'-b'')$		
26.45		$A'-A''$	$30' = 1800''$
30.0		log tan γ	8. 239
34.0		$\log(A'-A'')$	3. 255
38.5	12.05	colog $2(b'-b'')$	8. 907
42.4	12.40		
46.75	12.75	log d	0. 401
	12.40	d	2. 52

46. Inequality of Pivots.—When a spirit-level is used to determine the inclination of a line, such as the horizontal axis of a transit, its readings and the resulting inclination will be vitiated by any inequality which may exist in the diameter of the pivots upon which it rests. To test for such an inequality let the instrument be firmly mounted and the inclination, b', be measured with the level as shown in § 42; then lift the axis out of the wyes, turn it end for end, and replace it so that what was the east pivot shall now rest in the west wye. Again measure the inclination, b'', and repeat the levelings and reversals several times, so that any systematic difference which may exist between b' and b'' shall be well determined. We now put

$$i = \tfrac{1}{4}(b' - b''), \qquad (87)$$

where i is the correction for inequality of pivots, and find for the true inclination of the axis in the two positions,

$$b_1 = b' - i, \qquad b_2 = b'' + i. \tag{88}$$

The correction i should be carefully determined and applied to all measured values of the inclination.

47. The Theodolite. — This instrument, which is also called engineer's transit, altazimuth, universal instrument, etc., is one with whose general appearance and construction the student is supposed sufficiently familiar to recognize its close relationship with the coordinates of System I, altitude and azimuth. Trace out this relationship in Plates I, II, and III, which represent different types of this instrument. The line of sight (telescope) is a radius vector of undetermined length; the horizontal and vertical circles measure azimuths and altitudes, or zenith distances, and in an ideally perfect instrument the readings of these circles should be the true azimuth and altitude of the line of sight, or at most should differ from these only by a constant *index correction*.

It may readily be seen that among the conditions which must be satisfied in the construction of such an instrument are the following:

(1) The axes must be perpendicular to each other.

(2) The line of sight must be perpendicular to the horizontal axis.

(3) The vertical axis must be truly vertical.

Owing to unavoidable imperfections of mechanical

work it is not probable that any one of these conditions is exactly fulfilled in any given instrument, and they are therefore to be regarded as so many sources of error, whose effects may be made small by careful adjustment, but whose complete elimination must be sought in some other way; e.g., if the vertical axis is not truly vertical, we may determine as follows the means for correcting the effect of this error upon the measurement of altitudes.

48. Zenith Distances.—In Fig. 9 let HZ be the direc-

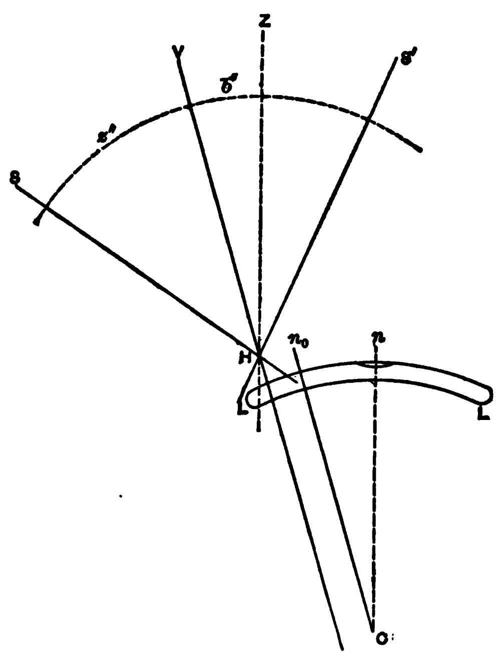


FIG. 9.—Measurement of Zenith Distances.

tion of the vertical; S, a point whose zenith distance, ZHS, is to be determined; HV, the projection of the vertical axis of the instrument upon the plane HZS;

and let r' denote the reading of the vertical circle when the line of sight is directed toward S. After reading r' let the instrument be turned about the vertical axis through an angle of 180°, bringing the line of sight into the position HS', and then let the telescope be turned about the horizontal axis until the line of sight again points at the object S, and let r'' be the reading of the vertical circle in this position. If the circle is numbered in quadrants, as is very common in small instruments, r' and r'' will be approximately the same number but with a graduation extending from 0° to 360°, as is here supposed, they will be widely different. From the figure, the angle SHS' is measured by the difference of these circle readings, $r'-r''$, and since $VHS=VHS'$, we have for the angular distance of the point S from V the equation,

$$VHS = z'' = \tfrac{1}{2}(r' - r''). \tag{89}$$

When two pointings of the telescope are made as above, the instrument is said to be *reversed* between them, and it is customary to designate its two positions as Circle Right and Circle Left, respectively, the reference being to the vertical circle of the instrument, which faces to the observer's right in the one position and to his left in the other. The student should note that the angle z'' is here determined quite independently of the adjustment of the verniers, which may be intended to read altitudes, zenith distances, or anything else, since the reversal eliminates all question of adjustment from the difference $r'-r''$, and is made for this purpose.

The true zenith distance of S is, however, not z'' but the angle,

$$ZHS = z'' + b'',$$

and b'' may be determined, as in § 42, from readings of the spirit-level, LL, attached to the instrument in such a way that its plane is parallel to the line of sight, HS. Such a level, i.e., one whose tube is perpendicular to the horizontal axis of the instrument, will be called the *altitude level* of the instrument.

A convenient method of taking into account the readings of the level-bubble by applying them directly to the circle readings instead of to the measured angles, is as follows: Let n_0 represent the reading of that point of the level scale through which passes that radius of the level which is parallel to the vertical axis, HV, Cn_0 in the figure, and let n denote the position of the middle of the bubble corresponding to the circle reading r'; i.e., since the bubble always stands at the highest point of its tube, n is the point exactly above the centre of curvature, C. It is evident from the figure that

$$b'' = (n - n_0)2d = (a + b - 2n_0)d, \qquad (90)$$

where d is the value of half a level division, and a and b are the actual scale readings of the ends of the bubble.

If the instrument had been, from the first, perfectly levelled we should not have obtained r' as the reading to the point S, but in place of r' a number either greater or less than it by the amount b''; and if, therefore, we apply to r' and r'' level corrections determined by the equation above given for b'', we shall reduce the read-

PLATE III.

A German Universal Instrument. Length of Horizontal Axis 12 inches. Approximate Cost $400.

[*To face p.* 114.]

ings to what they would have been for a perfectly levelled instrument, and therefore obtain the zenith distance of S immediately from the half difference of the corrected readings. Since any constant term which appears in the level correction will be eliminated from this difference of the corrected readings, $r'-r''$, we may substitute in Equation 90, in place of $2n_0$, any constant number whatever, e.g., zero, but it is usually convenient to take as this number S, the total number of divisions included in the level scale, since in the long run this will make the level corrections small. Making this substitution, we have finally,

$$\text{Level Correction} = \pm(a+b-S)d, \tag{91}$$

where the ambiguous sign depends upon the direction in which the numbers increase along the level scale, and may be determined, once for all, for a given instrument as follows: Two readings of the vertical circle of a certain instrument were taken to the same object, but with the instrument thrown out of level in such a way that the bubble stood at quite different parts of the scale in the two observations; e.g.:

Observation.	Bubble. a	b	Circle.	Level Corr.	Corrected r.
First.	2.0	25.8	91° 9′ 8′	+18″.7	91 9′ 26″.7
Second.	7.9	31.9	91 9 40	−12 .5	91 9 27 .5

The numerical values of the quantities above marked Level Corr. were computed from Equation (91) with an assumed value of $d=2''.6$, and since the effect of these corrections must be to bring the corrected circle readings into agreement, it is evident that the + sign must be used for the first observation and the − sign for the second. The whole number of divisions in the level scale being 35, the formula for this instrument becomes,

$$b'' = +2''.6\,[35-(a+b)].$$

A similar formula may be obtained for every instrument, and a table should be constructed from it, which with the argument $a+b$ will show the value of b'' for any given position of the bubble. Part of such a table is given below, and from it the level correction corresponding to any ordinary position of the bubble may be determined by inspection.

$a+b$	b''	$a+b$
30	$+13''.0-$	40
31	$+10\ .4-$	39
32	$+\ 7\ .8-$	38
33	$+\ 5\ .2-$	37
34	$+\ 2\ .6-$	36
35	$+\ 0\ .0-$	35

In the second observation given above we have $a+b=7.9+31.9=39.8$, and corresponding to this number we find, by interpolation from the table, $b''=-12''.5$.

The level formulæ thus derived show that if the bubble be brought to the same place in the tube, same values of a and b, both Circle R. and Circle L., the level correction will be eliminated from the difference $r'-r''$, and may therefore be neglected. To obtain the maximum precision however, the level should always be read and a correction applied to each circle reading, but even when this is done it is good practice to touch up the levelling screws after each reversal and bring the bubble back as near as may be to its first position, without, however, spending too much time in obtaining an accurate agreement.

In some instruments the level and verniers are attached to a frame (alidade) which admits of rotation about the horizontal axis without disturbing the direction of the line of sight. For such an instrument it may be shown that if, before reading the vernier, the frame be turned until the bubble stands at the middle of the scale, the resulting vernier readings are equiva-

lent to the corrected circle readings derived above, and therefore require no further correction for level error. This mechanical device, although convenient for some purposes, is of inferior accuracy.

49. **Effect of Errors of Adjustment.** — A geometrical investigation similar to the above may be made to show the effect of each source of instrumental error, but we shall find it more convenient to develop the combined effect of these errors through an analysis based upon Fig. 10, which represents a part of the celestial sphere,

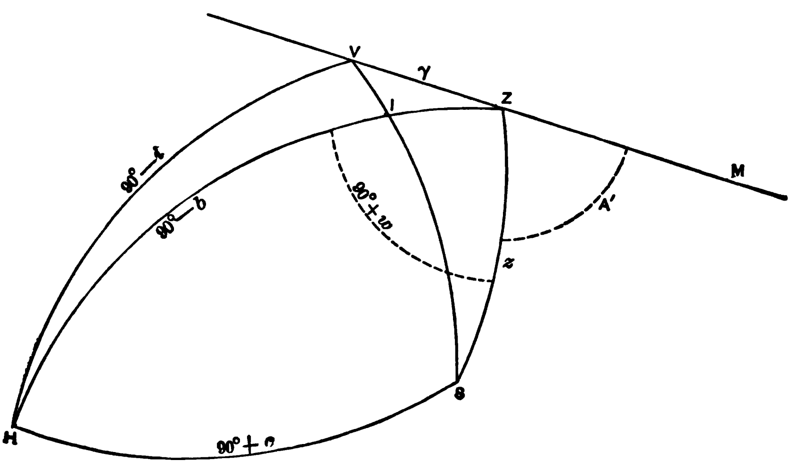


FIG. 10.—Theory of the Theodolite.

where Z is the zenith, V is the point in which the vertical axis of the instrument, when produced, cuts the sphere, H is the point of the sphere determined by the prolongation of the horizontal axis, and S is a star or other object whose azimuth and altitude are to be determined from readings of the horizontal and vertical circles of the instrument. The angles measured by means of these

circles lie in the planes of the circles, but just as the azimuth of a point is measured either by an arc of the horizon or by the corresponding spherical angle at the zenith, so the data furnished by the vernier readings may be regarded as spherical angles having their vertices respectively at V and H. Thus if r represent the reading of the vertical circle when the line of sight is directed toward S, and r_0 is the reading when this line is directed toward some point in the arc HV, the difference, $r-r_0$, measures the spherical angle VHS. Similarly, for the horizontal circle, by rotating the instrument about its vertical axis, H may be moved from its present position, corresponding to the reading R, into a new position falling upon the arc VM, and if R_1 be the circle reading in this position, we shall find that $R-R_1$ equals the spherical angle HVM. From these spherical angles, determined by the circle readings, it is required to find the true direction, $MZS=A'$, and the true zenith distance, $ZS=z$, of the star S.

It is evident from the figure that the arc $VH=90°-i$, measures the angle between the vertical and the horizontal axis of the instrument, and that i is therefore the error of adjustment of the axes, corresponding to Condition 1, § 47. Similarly, $HS = 90°+c$ measures the angle between the horizontal axis and the line of sight, and c is the error in the adjustment corresponding to Condition 2. Also, $VZ=\gamma$ is the error of level of the instrument, i.e., deviation of the vertical axis from the true vertical, corresponding to Condition 3. The arc $HZ=90°-b$ measures the angle that the horizontal axis makes with

the true vertical, and b is therefore the level error of this axis. Note that as the instrument is turned into different positions by rotation about the axes V and H, the quantities γ, i, and c remain unchanged and are therefore called *instrumental constants*, since they define the condition of the instrument with respect to its several adjustments. The level error, b, is sometimes included among these constants, but is not strictly one of them, since its value changes as the instrument is turned in azimuth.

We shall suppose the instrument to be so well adjusted that none of the instrumental constants exceeds $2'$, and H will then be so near the pole of the great circle VZM that we may assume without sensible error $HVM = HZM$ and, replacing these quantities by their equivalents, obtain

$$R - R_1 = (90^\circ + w) + A'$$

or

$$A' = R - (R_1 + 90^\circ) - w. \qquad (92)$$

The azimuth of S, reckoned from the true meridian instead of from the arc VM, differs from A' only by the substitution of another constant, the index correction of the horizontal circle, in place of $R_1 + 90^\circ$; and as this index correction must in any case be separately determined (see § 38), we may replace the constant term $R_1 + 90^\circ$ by R_0, the index correction referred to the true meridian, and we shall then have for the true azimuth of S,

$$A = R - R_0 - w. \qquad (93)$$

The auxiliary quantity w has thus far been defined only by means of Fig. 10, where the spherical angle HZS is labelled $90°+w$. To determine the value of w in terms of the instrumental constants, we have from the triangle HZS by means of Equations 4, the relation,

$$-\sin c = \sin b \cos z - \cos b \sin z \sin w,$$

which, since b and c are small quantities, is equivalent to

$$w = \frac{c}{\sin z} + \frac{b}{\tan z}, \tag{94}$$

or, replacing the zenith distance, z, by the star's altitude, h,

$$w = c \sec h + b \tan h. \tag{94*}$$

Since neither i nor γ enters into this equation, the effect of these errors must be taken into account through b, the inclination of the horizontal axis. This is to be determined with a spirit-level, and each circle reading, R, must be corrected for the particular inclination of the axis that corresponds to R. The factor $\tan h$ becomes zero for an object in the horizon, and for this special case the effect upon the azimuth readings of an error of level is zero. On the other hand, when the object to be observed is at a considerable elevation, e.g., the Pole Star in an azimuth determination, the factor, $\tan h$, becomes large and the effect of level error is magnified. It is in fact one of the chief sources of error in such determinations.

50. Determination of Errors of Adjustment.—The error above represented by c is called the *collimation*, and

its effect is usually to be eliminated through a reversal of the instrument. Since the angular distance of S from one end of the horizontal axis is $90° + c$, its distance from the other end must be $90° - c$, and as in the reversal these ends change places the effect of c must have one sign Circle R., and the opposite sign Circle L., and will therefore be eliminated from the mean of observations taken in both positions.

In precisely the same way it may be shown that the effect of i, error of adjustment of the axes, is eliminated from the mean of observations taken in the two positions, and wherever any considerable precision is required in azimuth observations or in the measurement of horizontal angles, the observer should not fail to make an equal number of pointings in each position of the instrument to secure this elimination of errors.

In the triangle HVZ the angle $HVZ = HZM$ is very nearly equal to $90° + A'$, and assuming this equality we find from this triangle,

$$\sin b = \sin i \cos \gamma - \cos i \sin \gamma \sin A', \tag{95}$$

which is equivalent to,

$$b = i - \gamma \sin A'. \tag{96}$$

The quantity $\gamma \sin A'$, which we shall represent hereafter by the symbol b', and which corresponds to the arc ZI of Fig. 10, is that component of the level error of the vertical axis, γ, which lies at right angles to the line of sight and which may therefore be determined from the readings of a level parallel to the horizontal

axis of the instrument. Such a level is called the *azimuth level*, and if resting upon the axis and capable of reversal (striding-level), it is most conveniently used to determine the level error of this axis, b. If fastened to the frame of the instrument and incapable of reversal, it may be used to determine, from bubble readings taken Circle R. and Circle L., the value of b' for the vertical axis, and corresponding to these two cases we shall have the following expressions of the level corrections to be applied to readings of the horizontal circle:

Striding-level,	$-b \tan h$.	Both Circle R. and Circle L.
Fixed Level,	$-b' \tan h$.	Mean of Circle R. and L.

If, as is usual, the graduation of the circle increases from left to right, b and b' are to be considered essentially positive when the high end of the horizontal axis has an azimuth 90° greater than the object S.

The student should not fail to note in connection with the use of a fixed azimuth level that if the bubble is brought to the same scale reading, Circle R. and Circle L., b' will be zero and the level error will be eliminated from the mean result.

A reversal furnishes a convenient method for determining or adjusting the collimation. For this purpose let R' and R'' be readings of the horizontal circle corresponding to observations of a fixed mark in or very near the horizon, made in the two positions of the instrument; then, from Equations 92 and 94,

$$2c = R' - R''. \tag{97}$$

To determine the error of adjustment of the axes, i, let the inclinations of the horizontal axis, b_1, b_2, be measured in two positions of the instrument differing 180° in azimuth, i.e., when Vernier A reads 0° and when it reads 180°. We shall then have, from Equation 96,

$$b_1 = i - \gamma \sin A',$$
$$b_2 = i - \gamma \sin (A' + 180°) = i + \gamma \sin A',$$

from which we obtain immediately

$$2i = b_1 + b_2. \tag{98}$$

If we call the inclinations b_1, b_2 positive when the circle end of the axis is too high, a positive value of i will indicate that the same end is too high, i.e., it makes too small an angle with the upward extension of the vertical axis.

The value of γ, which will seldom be required, may be found from four values of b determined at intervals of 90° in azimuth.

51. Additional Theorems. — By an analysis similar to that employed above, it may be shown from the triangles HSZ, HZV, of Fig. 10, that the errors b, c, and i have no appreciable influence upon observations of altitude or zenith distance. Indeed, it may be seen without formal analysis that when c, b, and i are small quantities, H is so nearly the pole of the circles ZS, VS, that these arcs are measured by the corresponding angles at H, i.e., by the readings of the vertical circle uncorrected for instrumental error. Since the error corresponding to γ is taken into account in the approximate analysis

of § 48, we may adopt as definitive the results there obtained. The correction b'' there determined is the arc VI of Fig. 10, i.e., it is the projection of γ upon the line of sight, VS.

The demonstration of the following theorems, which are of some consequence in the use of a theodolite, is left to the student.

1. If, as is quite common in engineer's transits, the vertical circle is graduated into quadrants instead of from 0° to 360°, observations of altitude should be made in the way already indicated, but in their reduction we shall have, in place of the formula for z'', the substitute,

$$h'' = \tfrac{1}{2}(r' + r''), \tag{99}$$

i.e., the mean of the readings gives directly the instrumental altitude.

2. The altitude level of such an instrument usually has the zero of its scale placed at the middle of the tube, and when such is the case readings of that end of the bubble nearest the objective end of the telescope should be marked o, and those of the end nearest the eyepiece should be called e; the formula for level correction then becomes,

$$b'' = (o - e)d. \tag{100}$$

3. A theodolite may be reversed by lifting the telescope from its supports, turning the axis end for end, and replacing it in the wyes in the changed position. This mode of reversal eliminates errors of level and collimation quite as well as does the one above described,

and also eliminates the inequality of pivots from the determination of b. It is therefore to be preferred when it can be conveniently practised.

52. **Errors Arising from the Circle Readings.** — Numerous errors of a class not considered above, creep into the results of observation through the circle readings, which may be vitiated in greater or less degree by:

(*a*) Defective graduation of the circle itself.

(*b*) The plane of the circle not being normal to the rotation axis.

(*c*) The circle not being truly centred upon the axis.

(*d*) The spaces on the vernier being too large or too small relative to those on the circle.

(*e*) Error of focussing (runs) in the reading microscopes.

etc. etc. etc.

The detailed study of these sources of error lies beyond the scope of the present work, but we note that in great part their effects may be eliminated by taking the mean of a considerable number of observations in which the circle readings are symmetrically distributed throughout the whole 360° of the graduation. Thus if an angle of 120° between objects A and B is measured three times and the circle turned 120° after each measurement so as to obtain the following system of readings:

	To A.	To B.	B−A.
Observation 1......	$0^\circ\ 0'\ 0''$	$120^\circ\ 0'\ 0''$	$120^\circ\ 0'\ 0''$
" 2......	120 0 0	240 0 0	120 0 0
" 3......	240 0 0	360 0 0	120 0 0

whatever graduation errors may affect the particular reading $120° 0' 00''$ will be eliminated from the mean value of $B-A$, since this reading enters into that mean once with a plus sign and once with a minus sign. If the required angle is small, e.g., $1°$, it will not be convenient to carry out the above programme of reading around the entire circle, but the elimination of errors may still be made by shifting the circle so that the readings to object A may be symmetrically distributed through the entire circumference, e.g., every $60°$ or every $30°$. For an instrument provided with two verniers or microscopes it will suffice to distribute the readings of each vernier over an arc of $180°$.

53. The Method of Repetitions. — A peculiar method of measuring horizontal angles may be adopted with advantage if, as is often the case, the instrument is provided with two motions in azimuth called, respectively, upper and lower, one of which produces a change in the vernier readings, while in the other, verniers and circle remain firmly clamped together and turn simultaneously, without change in the circle reading. Reverting to § 52, we may note that the circle readings $120°$, $240°$, there recorded, are quite unnecessary since, if the first reading, $0°$, be subtracted from the last one, $360°$, and the result divided by 3, we shall have as the value of the angle $120° 0' 0''$, which is precisely the same as the mean of the three values of $B-A$, and is all that that mean can furnish.

This process is called the *method of repetitions* and consists, essentially, in making a series of pointings upon

two objects between which an angle is to be measured, turning always from A to B upon the upper motion of the instrument and from B to A upon the lower motion, so that the vernier reading in the latter turning is not changed. A series of such pointings is called a *set* and the verniers need be read only for the first and last pointings of the set. If the initial and final readings be represented by R' and R'', and n be the number of pointings to each object contained in the set, we shall have, as shown above,

$$\text{Angle} = \frac{R' - R''}{n}. \tag{101}$$

It is often advantageous to reverse the instrument at the middle of a set, turning on the lower motion, and thus secure an additional elimination of instrumental errors.

The advantages of the method of repetitions are a saving of labor through the diminished number of vernier readings and, where the verniers are comparatively coarse, an increase of accuracy through the introduction of the divisor n into the value of the angle. The precision of a small instrument, such as an engineer's transit, may be considerably increased in this way, but for the larger instruments, provided with micrometer microscopes, experience shows that the best results are to be obtained by reading the microscopes after every pointing.

Where a horizontal angle between objects at very different altitudes is to be measured by the method of repetitions, as in an azimuth determination, an additional source of error requires careful attention, viz., the effect of a lack of parallelism between the axes corre-

sponding to the upper and lower motions of the instrument. To eliminate this error we proceed in the following manner: The axis of the lower motion should be made as nearly vertical as possible, and whatever may be the error of the upper axis it will produce no effect upon the final result if the number of repetitions is so chosen that the set extends through 360°; for in the successive turnings about the lower motion the upper axis has been made to describe a complete cone about the lower axis, and any error which may have been caused by a deflection to the east in one part of the set is balanced by the opposite error, caused by a deflection to the west, in another part, etc. If the angle to be measured is so small that the set cannot be made to extend through 360°, the following observing programme will also eliminate the error of the axis: Measure a set of any desired number of repetitions. When it is completed leave the instrument clamped at the last vernier reading, reverse about the lower motion and repeat the set in the opposite direction, i.e., beginning with the object last sighted upon and with approximately the vernier reading last obtained.

The level correction to the circle readings should be derived in the ordinary way, § 50, from readings of a level taken when the instrument is reversed about the lower axis.

54. Precepts for the Use of a Theodolite. — The experience of the principal geodetic surveys indicates that the following precepts should be observed in all precise work with a theodolite:

(1) An equal number of measurements should be made in each position of the instrument, Circle R. and Circle L.

(2) An equal number should be taken in each direction, i.e., the line of sight turned from right to left and from left to right.

(3) The position of the circle should be so shifted from time to time that the readings to *each* object are symmetrically distributed throughout the 360°.

(4) The observations should be made as rapidly as the observer can work without undue haste.

55. **The Sextant.** — A sextant consists essentially of two mirrors and a graduated arc of a circle, about 60°, for measuring the angle between the planes of the mirrors. The peculiar value of the instrument lies in the fact that it is light and portable, requires no fixed support, and may therefore be used for the measurement of angles at sea as well as on shore, and in any plane, vertical, horizontal, or inclined. For the purpose of description and analysis we suppose the sextant to be placed upon a table, with the plane of its arc horizontal, and we shall use the terms altitude, azimuth, etc., with reference to this special position of the instrument. The conclusions drawn from this consideration of the instrument apply equally when it is used in any other plane.

The essential parts of a sextant are indicated in Fig. 11 which should be compared with Plate IV. At the centre of the arc is a vertical axis carrying a vernier-arm, V, and also supporting one of the mirrors called the *index-glass*, I, whose plane is vertical, passes

nearly through the axis and rotates with the vernier-arm as the latter is turned in azimuth. At one side of the sextant frame is the other mirror, H, called the *horizon-glass*, with its plane vertical and fixed parallel to that radius of the graduated arc which is numbered 0°. Only the lower half of the horizon-glass is silvered, the upper half is left transparent. A telescope, T, is mounted on the side of the frame opposite to the horizon-glass and has its line of sight directed toward the latter. From Fig. 11 it may be seen that an observer looking

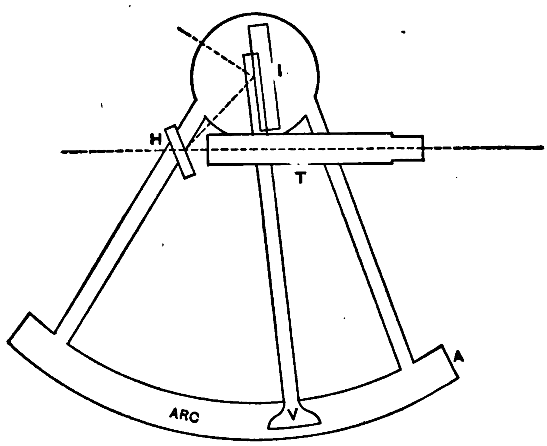
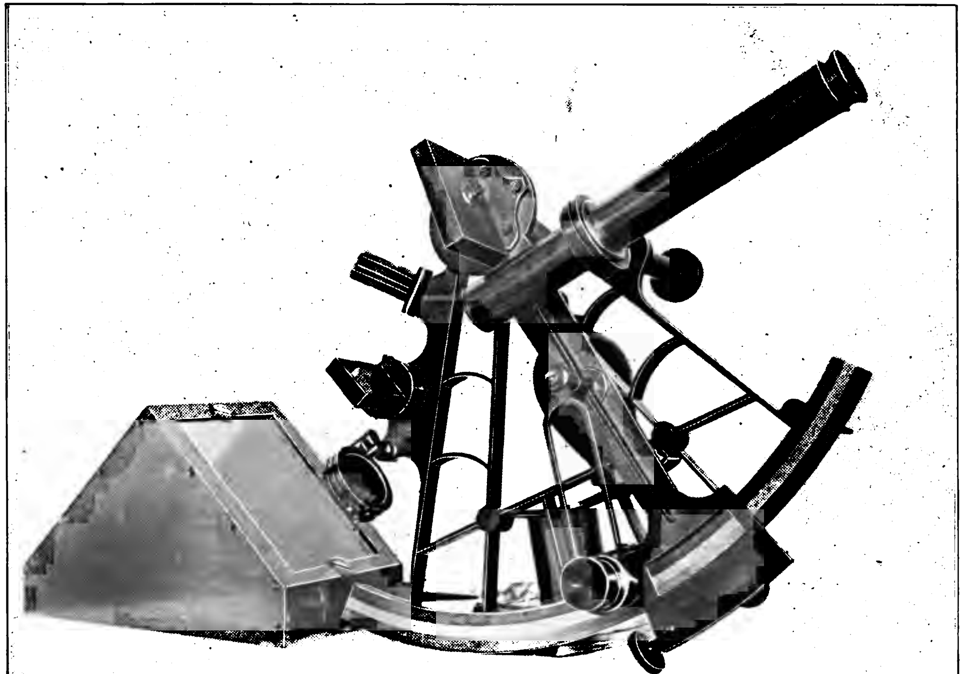


FIG. 11.—Elements of a Sextant.

into the telescope and through the unsilvered upper half of the horizon-glass will see that part of the horizon toward which the telescope is directed, and will also see superposed upon it a view of another part of the horizon reflected from the index-glass to the silvered half of the horizon-glass, and from this again reflected into the telescope. This part of the horizon is said to be seen



A Sextant and Artificial Horizon. Radius of Graduated Arc 7 inches. Approximate Cost $140.

[*To face p.* 130.]

reflected, while the part seen through the horizon-glass is observed *direct*. Any reflected image which is superposed upon a direct image is said to be *in contact* with the latter, and we shall represent these images as seen in the telescope, by I and H respectively.

By turning the index-glass in azimuth, different parts of the horizon may be reflected into the telescope, and since the rays of light incident upon and reflected from the mirror make equal angles with its surface, it is apparent that for every 1° that the mirror is turned, the azimuth of the point reflected into the telescope will be changed by 2°. There may be found by trial a setting of the index-glass at which both a direct and a reflected image of the same object may be seen simultaneously and may be made to pass one over the other as the vernier-arm is slightly turned. Let R_0 denote the vernier reading when these images are brought into contact, and let R be the reading at which any other object, I, is brought into contact with the H just observed; then it appears from the above that the difference of azimuth between I and H is twice the angle included between R_0 and R. On account of this multiplier, 2, each half-degree of the sextant arc is numbered as if it were a whole degree, and we have, therefore, for the difference of azimuth,

$$H - I = R - R_0. \tag{102}$$

The $-R_0$ which appears in this equation is called the *index correction*, and it should be observed that, owing to the angle subtended at the object H by the space

separating the index and horizon glasses, the reading R_0 will depend upon the distance of H from the instrument. If it is near at hand, less than three miles, R_0 should be determined as above and the axis of rotation of the index-glass should be centred over the point at which it is desired the vertex of the measured angle should fall. If the objects are very remote, all question of the exact position of the vertex is eliminated, and a mode of determining the index correction given hereafter will be found more convenient than the above.

It may now be seen that the following conditions must be satisfied in order that the Equation 102, given above, shall furnish the true value of the angle between H and I:

(*a*) The rotation axis and the plane of the index-glass must be perpendicular to the plane of the graduated arc. If they are not perpendicular, this arc cannot accurately measure the amount of rotation of the mirror.

(*b*) The horizon-glass must be perpendicular to the plane of the arc. If it is not perpendicular, the direct and reflected images of H cannot be brought into contact, but one will pass above or below the other as the vernier-arm is turned.

(*c*) The objects H and I must lie in the sextant horizon, for otherwise the difference of their azimuths would not be the true angle between them. The sextant horizon must here be understood to mean the plane of the graduated arc, and this condition will be satisfied if the sextant is so held during the observation that this plane passes through the objects H and I.

56. Adjustments of the Sextant.—(A) *The Index-glass.*—Take the telescope out from its support, set it on end at any part of the arc, and turn the index-glass until its plane passes a little to one side of the telescope. By holding the eye a little to the right of the line joining the index-glass to the telescope a reflected image of the telescope may be seen simultaneously with a direct view of it, and these two images should be parallel, provided the telescope stands normal to the plane of the arc. Any error in this last condition may be eliminated by turning the telescope 180° about its own axis and repeating the test. No adjusting-screws are provided for the index-glass, but it may be adjusted, if necessary, by removing it from its frame and filing down the bearing-points against which it is held.

(B) *The Horizon-glass.*—Bring the direct and reflected images of a distant object into contact if possible. If this cannot be done, bring them near together and tilt the horizon-glass by means of its adjusting screws until by turning the vernier-arm the images can be made to coincide.

(C) *The Telescope.*—To enable the observer to make the plane of the sextant pass through the objects H and I it is customary to place in the eyepiece of the telescope a pair of coarse threads which should be set parallel to the plane of the sextant. By means of its adjusting screws the telescope should be tilted up or down until the line of sight passing midway between these threads is parallel to the plane of the sextant. If the objects H and I are brought midway between these

threads when contact between them is made, they will lie in the plane of the sextant as required. To determine if the telescope is properly tilted, select two well-defined objects about 120° apart, and bring them into contact when the sextant is so held that they are both seen in the upper part of the field of view. Then shift the position of the sextant plane so as to bring the objects to the lower part of the field and note whether they remain in contact or appear separated; if they are appreciably separated the telescope requires further adjustment.

57. Outstanding Errors of the Sextant.—The methods of adjustment above described are only approximate, and the readings of the instrument will be affected by whatever error remains in the adjustment. In general the effect of these errors will be small for small angles, but will increase rapidly with the magnitude of the angle measured, and the adjustments should be made correct to within 10′ if the resulting errors for an angle of 90° are to be insensible.

However carefully these adjustments are made there will remain a source of error which cannot be removed by adjustment, but whose effect must be determined and applied as a correction to the readings if the maximum attainable precision of the instrument is required. It is assumed above that the centre of the graduated arc falls exactly at the centre of motion of the index-glass, but the maker is seldom able to secure this exact agreement, and without it the readings of the vernier are not an accurate measure of the amount of rotation of the

mirror. The effect of this error, which is called *eccentricity*, combined with the effect of all other outstanding errors of the instrument, is best determined by carefully measuring with it a set of known angles of different magnitudes, from o° to the largest one possible, and treating the difference between the measured value and the true value of each angle as a correction to the corresponding reading of the sextant. These corrections may be plotted as ordinates with the sextant readings as abscissas and a curve drawn, from which intermediate values of the correction may be read. The length of the arc joining two stars whose right ascensions and declinations are given, may be computed and used as a known angle for this purpose, provided the effect of refraction in altering this distance is duly taken into account; or if a distant part of the horizon can be seen, a set of angles may be measured with a good theodolite for comparison with the sextant results.

58. Index Correction.—Since the value of the index correction for very distant objects is constant so long as the adjustments of the sextant remain unchanged, it may be determined from special observations made for this purpose, but the determination should be frequently repeated since the adjustment is easily disturbed. Let a shade-glass be placed over the eye end of the telescope and the direct and reflected images of the sun brought into contact, externally tangent to each other, in each of the two possible positions, H first right, then left of I. The mean of the corresponding sextant readings will be the required value of R_0. Since the index

correction enters into the value of every measured angle, it should be carefully determined from several settings, as in the following example:

DOUBLE DIAMETER OF SUN FOR INDEX CORRECTION.

Caroline Island, April 22, 1883.

Observer, W. U.

On Arc.	Off Arc.	Reduction.
0° 26′ 0″	359° 22′ 5″	R_0 = 359° 53′ 54″
26 0	22 0	i = +6 6
26 5	22 0	
26 0	21 40	$4S$ = 1 4 0
25 40	21 50	S = 0 16 0
25 40	21 50	Almanac = 0 15 56.4
360° 25′ 54″	359° 21′ 54″	

In place of subtracting R_0 from each subsequent reading of the instrument it is in this case more convenient to employ the quantity $i = 360° - R_0$ as a correction to be added. The readings "Off Arc" were taken on the supplementary arc to the right of the 0°, and the student should note that the resulting R_0 falls off the arc. Referring to the position of R_0, the sign of the index correction may be determined from the sailor's rule: When it (R_0) is on it's off (*i subtractive*), and when it (R_0) is off it's on (*i additive*). The values of the sun's semi-diameter furnished by the observations and given in the almanac are shown above for comparison.

59. **Artificial Horizon.** — Altitudes may be measured with a sextant either from the natural (sea) horizon or from an artificial horizon, one form of which is a shallow box containing mercury, covered by a glass roof to protect it from wind. The reflecting surface of the liquid is improved by adding to it a little tin-foil and removing

the resulting scum (oxide) with the edge of a card. The reflected image of the sun or star lies as much below the true horizon as the real object is above it, and if the angle between the two is measured with the sextant it gives at once the double altitude of the body, subject to correction for index error, etc. See § 29 for an example.

60. Precepts for the Use of a Sextant.—1. Keep your fingers off the graduation. It tarnishes readily.

2. Focus the telescope with great care so as to secure sharply defined images.

3. Make the direct and reflected images equally bright, by moving the telescope to or from the plane of the sextant with the adjusting-screw provided for this purpose.

4. Bring the images into contact midway between the guide-threads.

5. Don't try to hold the images still in the field of view. Give the reflected image a regular oscillating motion by twisting the wrist, and note its relation to the direct image as it swings by.

6. In observing the sun take an equal number of observations on each limb (edge).

7. Take an equal number of observations in each position of the horizon roof, direct and reversed.

8. Determine the index correction as carefully as the angle which you wish to measure.

9. Whenever possible use a shade-glass over the eyepiece instead of those attached to the sextant frame.

10. Work as rapidly as you can without hurrying.

61. Chronometers.—This section will be confined to a consideration of the proper care and use of timepieces. For an account of their mechanical construction see the article Watches in the Encyclopædia Britannica.

A chronometer is a large and finely constructed watch, whose face, hands, and train (wheels) are to be considered as a mechanical device for automatically counting and registering the vibrations of a steel helix, called the *balance-spring*. In most chronometers this spring makes one complete vibration every half second, producing a *beat* (tick) of the chronometer and a forward movement of the seconds hand through $0^s.5$. This spring may vibrate too slow or too fast, thus producing a rate of the chronometer, and it is practically convenient that this rate should be small, but the real test of excellence in a timepiece is not the magnitude of its rate, but its uniformity of rate from day to day.

In order that the rate of a chronometer shall remain constant, every precaution must be taken against disturbing the balance-spring, and most of the following precepts for the treatment of a chronometer have reference to this condition. Of the various mechanical disturbances to which it is subject, experience shows that a quick rotary motion about the axis of the balance-spring is the most injurious. According to the chronometer makers a single quick motion of this kind through half a turn and back may change the chronometer correction several seconds and so disturb the rate that it will not resume its normal value for hours or even days.

A chronometer is usually supported in gimbals and

should be allowed to swing freely in them when at rest, in order that it may assume a vertical position; but when carried about, the gimbals should be locked since the oscillations that would otherwise be imparted to the balance-spring are more injurious to the rate than the isolated shocks that it may receive when firmly held in one position. A chronometer should be kept in a dry place, not exposed to magnetic influences. If possible it should always rest in the same azimuth, e.g., the zero of the dial always pointing north. It should be wound at regular intervals, and its temperature should be kept as nearly uniform as possible. The average chronometer runs best at a temperature near 70° Fahr.

62. Comparison of Chronometers. — A problem of frequent recurrence is the comparison of one chronometer with another, e.g., in order to determine the correction of one from the known value of ΔT for the other. This comparison consists in noting the time indicated by one chronometer at a given time shown by the other, and presents little difficulty when no greater accuracy than the nearest half-second is required. If the comparison is to be made correct to the nearest $0^s.1$, the method of coincident beats may be employed if one of the chronometers keeps sidereal and the other solar time.

Since sidereal time gains 236 seconds per day upon mean solar time and the chronometers beat half-seconds, there will be 472 epochs during a day, at which the chronometers beat in unison, i.e., a coincidence of the beats occurs every three minutes throughout the day, and if the comparison be made at one of these coinci-

dences by noting by each chronometer its indicated time when the beats are coincident, no fractions of a second need be determined and the comparison can be made correct within one or two hundredths of a second.

This mode of comparison is illustrated in the following example of the comparison of two mean-time clocks, *M* and *F*, with each other by comparing each with a sidereal clock designated *H*.

OBSERVED TIMES OF COINCIDENT BEATS.

M	19^h	30^m	49^s	*F*	19^h	34^m	55^s
H	10	42	1	*H*	10	46	0

The interval between the coincidences, as measured by *H*, is $3^m\ 59^s$ (sidereal), and this interval reduced to mean solar units and added to *M*, or subtracted from *F*, gives a comparison between the mean-time clocks as follows:

M	19^h	34^m	$47^s.35$	19^h	30^m	$49^s.00$
F	19	34	55 .00	19	30	56 .65,

either form showing that *F* was $7^s.65$ faster than *M*.

Every observer should acquire the ability to "carry the beat" of a chronometer, i.e., to listen to and count the beats while attending to something else, since nearly all observations in which it is required to note the time of an event, e.g., the transit of a star over a thread, require this ability unless special mechanical devices, such as a chronograph, are employed. (See § 79.)

CHAPTER VIII.

ACCURATE DETERMINATIONS.

63. General Principles. — Where a high degree of precision is desired in the results of observation, the purely instrumental sources of error that have been examined in the preceding chapter must be eliminated by the methods there shown, or by others equivalent to them. But this alone is not sufficient, and we note, for example, that an instrument taken from a warm place and set up in a cold one undergoes a process of cooling and contraction of its parts that, while in progress, renders the errors of adjustment variable quantities, whose effects cannot be represented by the formulæ derived for the case of "instrumental constants." We have therefore as a rule to be carefully observed when precision is required: *Let the instrument be set up and levelled in the place where it is to be used, at least half an hour before observations are commenced.* Let the surroundings of the instrument during this period be as nearly as possible like those under which the observations are to be made, i.e., shutters open, lamps lighted, etc. As a corollary to this rule we have the further precept that during the progress of the observations the

observer and his lamp should be kept away from the instrument as much as possible.

There is large room for the display of good judgment in the selection of stars to be observed for a given purpose, such as the determination of time or azimuth, and precepts bearing upon this choice, both with reference to the precision of the observations themselves and to the elimination of errors in the right ascensions and declinations of the stars as furnished by the almanac, are given in the following sections.

Whenever observations are to be made upon a considerable number of different stars, as in determinations of time and latitude, an *observing list* should be prepared in advance, giving the names and magnitudes of the stars, arranged in the order in which they are to be observed, and giving also such data as may be required for finding them with the given instrument, e.g., their right ascensions, declinations, zenith distances, etc. Also, a form should be prepared in which to record the observations, each figure that is to be written down as a part of the record having its proper place allotted it. This place must be filled up before the observation is complete, and the presence of an unfilled space in the form is to be considered as a reminder that something remains to be done.

64. Time by Equal Altitudes. — The best method of determining time involves the use of a transit instrument (see Chapter IX), but an excellent time determination may be made with a theodolite, zenith telescope, or sextant by the method of equal altitudes, as follows:

We note the chronometer time, T_1, at which a star west of the meridian reaches the zenith distance z_1 and the time, T_2, at which another star, east of the meridian, reaches a zenith distance, z_2, which differs as little as possible from z_1. In sextant observing it is customary to assume that if the sextant is set to the same reading in the two observations we shall have $z_1 = z_2$. For an instrument of the other type (theodolite) the telescope must be left firmly clamped in altitude as it is turned from one object to the other, and any slight change in the altitude of the line of sight must be carefully determined from readings of the altitude level of the instrument. If the bubble changes its position in the level-tube when the latter is turned from the first to the second star, it should be brought back to its original place by the levelling screws of the instrument, but the angle between the telescope and level-tube must not be altered. If the instrument is provided with an azimuth circle, it will be well to note its readings, R_1 and R_2, corresponding to the observed T_1 and T_2.

For the reduction of the observations we take from the formulæ for transformation of coordinates, § 14, the equations

$$\begin{aligned} \cos z_1 &= \sin\phi \sin\delta_1 + \cos\phi \cos\delta_1 \cos t_1, \\ \cos z_2 &= \sin\phi \sin\delta_2 + \cos\phi \cos\delta_2 \cos t_2, \end{aligned} \qquad (103)$$

and in these relations if we could assume,

$$z_1 = z_2, \quad \delta_1 = \delta_2,$$

we should have at once,

$$\cos t_1 = \cos t_2 \quad \text{and} \quad t_1 = -t_2.$$

From this last relation we obtain,

$$(T_1+\Delta T)-a_1=a_2-(T_2+\Delta T),$$

and solving this for ΔT, find,

$$\Delta T=\tfrac{1}{2}(a_1+a_2)-\tfrac{1}{2}(T_1+T_2). \qquad (104)$$

This ideal case may be realized in practice by observing the times at which a given star comes to equal zenith distances on opposite sides of the meridian, i.e., before and after its culmination, but this may involve a delay of several hours between the observations, and it will usually be more convenient and expeditious to observe in quick succession two stars of nearly equal declination but widely different right ascension, one east and the other west of the meridian, or the sun, A.M. and P.M.

To adapt Equation 104 to this case we assume six new quantities, z, B, δ, D, t, and L, defined by the following relations:

$$\begin{array}{lll} z+B=z_1, & \delta+D=\delta_1, & t+L=t_1, \\ z-B=z_2, & \delta-D=\delta_2, & t-L=t_2, \end{array} \qquad (105)$$

and from the last pair of these equations we obtain, by the method followed in deriving Equation 104,

$$\Delta T=\tfrac{1}{2}(a_1+a_2)-\tfrac{1}{2}(T_1+T_2)+L. \qquad (106)$$

To determine the value of L we introduce into Equations 103 the quantities defined in Equations 105, and subtracting the first of these transformed equations from the second, obtain the rigorous relation,

$$\begin{aligned}\sin z \sin B = &-\sin\phi\cos\delta\sin D \\ &+\cos\phi\sin\delta\sin D\cos t\cos L \\ &+\cos\phi\cos\delta\cos D\sin t\sin L. \qquad (107)\end{aligned}$$

This equation is quite too cumbrous for use, but if in the plan and execution of the observations care is taken to make B and D small quantities whose cubes and higher powers may be neglected, it is readily reduced to the simpler form,

$$L = \frac{\tan \phi}{\sin t} D - \frac{\tan \delta}{\tan t} D + \frac{B}{\cos \phi \sin A}. \qquad (108)$$

From Equations 105 we find for use here,

$$\begin{aligned} B &= \tfrac{1}{2}(z_1 - z_2), \quad \delta = \tfrac{1}{2}(\delta_1 + \delta_2), \\ D &= \tfrac{1}{2}(\delta_1 - \delta_2), \quad t = \tfrac{1}{2}(a_2 - a_1) - \tfrac{1}{2}(T_2 - T_1). \end{aligned} \qquad (109)$$

It appears from these relations that the quantity B is half the change of zenith distance suffered by the line of sight in passing from one star to the other, and this change should be measured with all possible care by means of the altitude level of the instrument. If we represent by b the observed displacement of the bubble between the two observations and by d the value of half a level division, we shall have

$$B = \pm bd, \qquad (110)$$

where the positive sign is to be used when the bubble stands nearer to the objective end of the telescope at the eastern than at the western observation. The value of B is required in seconds of time, and it will therefore be convenient to express d in terms of the same unit instead of in seconds of arc.

The declination factor, D, should also be expressed in seconds of time, and since declinations are usually given in arc, we reduce the difference $\delta_1 - \delta_2$ to seconds

of arc, and dividing this by 15 obtain in terms of the required unit

$$D = \tfrac{1}{30}(\delta_1 - \delta_2)''. \qquad (111)$$

65. **Example.**—*Time by Equal Altitudes.*—The following example illustrates the application of these equations to the reduction of observations made with an engineer's transit provided with stadia threads, over which the star's vertical transits were observed, the instrument being turned between times so that the transit over the horizontal thread should always occur near its intersection with the vertical thread. The three terms contained in the value of L (Equation 108) are here represented by the symbols L_1, L_2, L_3.

EQUAL ALTITUDES FOR TIME.

Thursday, April 30, 1896.

At Brick Pier. Instrument, Heyde. Observer, C.

Star	α Orionis	α Serpentis		
Obs'd T	$10^h\ 41^m\ 9^s.6$	$10^h\ 50^m\ 42^s.1$	$\frac{1}{2}(T_1+T_2)$	$10^h\ 45^m\ 55^s.8$
	41 37.2	50 14.2		55.7
	42 5.2	49 45.3		55.2
Level*	19.7 3.3	18.8 2.6	−Mean	−10 45 55.57
R	83° 51′	277° 23′	L	+1 12.11
δ	+7 23 18.8	+6 44 51.3	$\frac{1}{2}(\alpha_1+\alpha_2)$	+10 44 22.24
α	$5^h\ 49^m\ 33^s.06$	$15^h\ 39^m\ 11^s.42$	ΔT	−0 21.22

* The end of the bubble nearer the objective is recorded first. $d = 3''.8 = 0^s.25$.

$\frac{1}{2}(\alpha_2-\alpha_1)$	$4^h\ 54^m\ 49^s$	$\delta_1-\delta_2$	$+0°\ 38'\ 27''.5$	b_1-b_2	$-0.8\,d$
$-\frac{1}{2}(T_2-T_1)$	−4 18	D	$+76^s.92$	B	9.3010n
t (time)	4 50 31	tan ϕ	9.9709	sec ϕ	0.1364
t (arc)	72° 38′	cosec t	0.0203	cosec A	0.0032
δ	7 4	D	1.8860	log L_3	9.4406n
ϕ	43 5	cot t	9.4952		
$A = \frac{1}{2}(R_1-R_2)$	83 14	−tan δ	9.0933n	L_1	+75.37
		log L_1	1.8772	L_2	− 2 98
		log L_2	0.4745n	L_3	− 0.28

For the sake of illustration the reductions in the preceding example are carried to hundredths of a second of time, but this is a quantity quite inappreciable in the telescope of an engineer's transit, and with such an instrument, or with a sextant, it will usually be sufficient to carry the reductions to tenths of seconds only. Corresponding to this degree of accuracy the difference of declination of the stars may be as great as two or three degrees without the introduction of sensible error into the results by reason of the approximate character of the reduction formulæ. The difference should not exceed one half of this amount if hundredths of seconds are to be taken into account.

66. Observing List.—Without transgressing these rather narrow limits for $\delta_1 - \delta_2$, a considerable number of suitable pairs of stars may be selected from the almanac, as is illustrated by the short observing list given below, and such a list should be prepared for the particular time and place at which observations are to be made. At least one of the stars in each pair should be a bright one, easily recognized and found with the telescope by sighting over its tube. The second star of the pair, even though much fainter, may be readily found by the method given below.

In the selection of pairs of stars care should be taken to secure those that are as near as may be to the prime vertical at the time when their altitudes are equal, since the motion in altitude is then most rapid and most accurately observed. The analytical expression for this condition is

$$\tan \tfrac{1}{2}(\delta_1 + \delta_2) = \tan \phi \cos \tfrac{1}{2}(\alpha_1 - \alpha_2)\,; \qquad (112)$$

and if this equation is satisfied by the coordinates of any two stars that differ but little in declination, these stars will be near the prime vertical at the instant when their altitudes are equal. But this condition should not be too rigorously insisted upon, and even considerable deviations from it may be permitted in order to secure a suitable number of bright stars.

Having chosen a pair of stars, we may determine as follows the sidereal time, θ, at which their altitudes will be equal: In Equation 106 we put $\Delta T = 0$, $T_1 = T_2 = \theta$, and obtain

$$\theta = \tfrac{1}{2}(\alpha_1 + \alpha_2) + L, \qquad (113)$$

where the value of L is to be derived from Equation 108, omitting the term in B. It will usually be convenient to observe the first star about five minutes before the computed time, θ.

Finding the Faint Star.—If the two stars have equal declinations, their azimuths at the instant of equal altitudes will be numerically equal but of opposite sign, i.e., $A_1+A_2=0$, while if their declinations, differ slightly, there will be a small difference in the azimuths which will transform this equation into

$$A_1+A_2+dA_2=0. \tag{114}$$

To determine the value of dA_2 in this equation we obtain from the astronomical triangle the relation (Equation 15)

$$\sin\delta=\cos z\sin\phi-\sin z\cos\phi\cos A, \tag{115}$$

and differentiating this, treating ϕ and z as constants, we find as the change of azimuth of the second star produced by a small change of declination,

$$\cos\delta\,d\delta=\cos\phi\sin z\sin A\,dA=\cos\phi\cos\delta\sin t\,dA, \tag{116}$$

from which,

$$dA=\frac{d\delta}{\cos\phi\sin t}. \tag{117}$$

Let R_1, R_2, R_0 represent respectively the readings of the horizontal circle when the telescope is directed to the western star, to the eastern star, and to the meridian; we shall then have

$$A_1=R_1-R_0, \qquad A_2=R_2-R_0, \tag{118}$$

and substituting in Equation 114 these relations together with the approximate values,

$$d\delta=\delta_2-\delta_1, \qquad t=\tfrac{1}{2}(a_2-a_1), \tag{119}$$

we obtain

$$R_1+R_2=2R_0+\frac{\delta_2-\delta_1}{\cos\phi\sin\frac{1}{2}(a_2-a_1)}. \tag{120}$$

The last term in this expression, computed for $\phi=43°$, is tabulated in the observing list under the heading ΔR, and by means of it and the reading R_1 to the first star of a pair, the reading of the horizontal circle, R_2, may be found at which the instrument should be set and the arrival of the second star in the field awaited at a time as much after the computed θ as the first observed time was earlier than θ. For convenience' sake orient the instrument and make $R_0=0$. As a control upon the sign of ΔR, note that the star that has the larger declination must be the farther from the south point.

TIME BY EQUAL ALTITUDES.

Partial Observing List for $\phi = 43°$.

Stars.	Mag.	R. A.	Dec.	θ.	ΔR.
		h. m.	° ′	h. m.	° ′
α Orionis.	0.9	5 50	7 23	10 44	−0 54
α Serpentis.	2.7	15 39	6 45		
α Can. Min.	0.5	7 34	5 29	11 34	+2 0
α Serpentis.	2.3	15 39	6 45		
β Geminorum.	1.2	7 39	28 16	12 43	−0 41
μ Herculis.	3.5	17 43	27 47		
α Leonis.	1.3	10 3	12 28	13 46	+0 16
α Ophiuchi.	2.2	17 30	12 38		
α Aquilæ.	0.9	19 46	8 36	14 44	−2 28
o Leonis.	3.8	9 36	10 21		

67. Precise Azimuths. — The azimuth of a terrestrial line, e.g., the line joining the centre of a theodolite to a distant mark, may be determined by measuring the difference of azimuth, D, between the mark and a star at an observed time, T. From the observed time and the right ascension of the star its hour angle, t, may be derived, and from Equations 14 we obtain, by division and introduction of the auxiliary quantities,

$$g = \cot \delta \sec \phi, \qquad k = \cot \delta \tan \phi, \tag{121}$$

the relation

$$-\tan A = \frac{g \sin t}{1 - k \cos t}, \tag{122}$$

from which the true azimuth of the star at the time of observation is readily computed. The azimuth of the mark is then

$$A' = A + D, \tag{123}$$

where D is assumed to be measured from the star toward the east.

The precision of A' depends equally upon D and A, and through A it depends upon the assumed latitude, declination, right ascension, and chronometer correction that are employed in the computation. The observations should therefore be planned with reference to eliminating whatever minute error may exist in any of these data, and to overcoming, by the methods indicated below and in § 54, the effect of instrumental errors upon the measured angle D.

Errors in the Assumed Data.—The effect of these errors may be greatly diminished by selecting for observation a star very near the pole of the heavens, since the factor g is thus made small, and such a star, e.g. Polaris, should always be chosen. If the chronometer correction is well determined, the observations may be made at any convenient hour, whether near elongation or not. As a guide to the required precision in ΔT we note that for observations of Polaris within the limits of the United States an error of 2^s in the time will in no case produce in the computed azimuth an error greater than $1''$.

If the highest precision is required, the star should be observed at two points of its diurnal path which are diametrically opposite to each other, i.e., there should be two groups of observations separated by an interval of twelve hours, or some odd multiple of twelve hours. Errors in ϕ, δ, and α will then be almost perfectly eliminated, and there will also be eliminated any systematic personal error of observation depending upon the direction of the star's apparent motion, such as is sometimes found to exist in the work of even the best observers.

A similar but less complete elimination of errors may be obtained from observations made at a single epoch if these are equally divided between stars upon opposite sides of the pole and equidistant from it. Examples of pairs of stars which approximately fulfil this condition are Polaris and 6 Ursæ Minoris; 51 H. Cephei and δ Ursæ Minoris.

The angle D may be measured with either a repeating or a non-repeating (direction) instrument, and the student should observe the following respects in which their use differs: For a repeating instrument the azimuth level should be used to determine the inclination of the vertical axis corresponding to the lower motion of the instrument. For a non-repeating instrument the inclination to be determined is that of the horizontal axis. In both cases the bubble readings are to be taken when the line of sight is directed toward the star and also when it is turned toward the mark, unless the latter has a zenith distance of 90°, in which case erroneous levelling will not affect the readings to it.

With any type of instrument the horizontal circle is to be turned in its own plane from time to time during the observations, so that the vernier or microscope readings shall be symmetrically distributed throughout the entire 360° of the graduation; e.g., for an instrument with two microscopes let one ninth of the total number of observations be made with the circle reading to the mark approximately 0°, another ninth with the circle reading 20°, 40°, 60°, etc. But see § 53 for the peculiar

manner in which the circle settings should be changed in the case of a repeating instrument.

Level Corrections.—The correction for level error is to be applied to each circle reading as shown in § 50, but for observations made by the method of repetitions the level correction, $b' \tan h$, there given for the reading to the star, must be multiplied by n, the number of pointings in a set, since the difference of the corrected readings to star and mark is to be divided by n in order to obtain the measured angle. It will usually be expedient to arrange the form of record of the observations so that the level corrections may be applied and the angles worked out in the record book.

68. Reduction of the Observations.—After the hour angles have been formed from the relation $t = T + \Delta T - \alpha$, and the constants g and k computed with the known declination and latitude, the computation of A presents no difficulties, but it may be considerably abbreviated through the use of Albrecht's Tables (reproduced in Appendix VII, Annual Report U. S. Coast and Geodetic Survey, 1897–98), which with the argument log x give the logarithm of $\frac{1}{1 - x}$. Calling this last factor F, and putting $k \cos t = x$, Equation 122 assumes the very simple form

$$-\tan A = gF \sin t. \qquad (124)$$

In the absence of special tables for F its value may be readily obtained from an ordinary table of addition and subtraction logarithms as follows: Representing

by A and B the argument and function in such a table,* i.e., $A = \log x$, $B = \log(1+x)$, we have, whenever $\cos t$ is negative, $A = \log(k \cos t)$, $\log F = -B$. When $\cos t$ is positive, we use the development

$$\frac{1}{1-x} = (1+x)(1+x^2)(1+x^4)(1+x^8), \text{ etc.,}$$

and interpolating from the table of addition logarithms the values of B_1, B_2, B_4, etc., corresponding to the arguments x, x^2, x^4, we find

$$\log F = B_1 + B_2 + B_4 + \text{etc.} \tag{125}$$

For observations of Polaris made within the limits of the United States it will never be necessary to use more than the first two terms of this series, e.g., corresponding to this case the greatest possible value of $k \cos t$ furnishes $\log x$ and the several values of B given below:

$\log x$	8.39386	B_1	0.0106248
$\log x^2$	6.7877	B_2	2663
$\log x^4$	3.575	B_4	2
		$\log F$	0.0108913

In ordinary practice the value of $\log F$ will be required to only six places of decimals, and $B_1 + B_2$ furnishes this degree of precision,

Where the highest degree of precision is sought, it is customary in the reduction of the observations to compute for each observed time the corresponding value of A, but this process may be very greatly abridged by treating the mean of a considerable number of observations as a single observation made at T_0, the mean of the recorded times. The azimuth A_0 computed from T_0 will not correspond exactly to the observations, but the correction required on this account is readily obtained.

* Do not confound this use of A with its wholly different meaning in Equation 124.

We may develop by Taylor's Formula the relation between azimuth and time in the form,

$$A = A_0 + f'(A_0)(T - T_0) + \tfrac{1}{2}f''(A_0)(T - T_0)^2 + \text{etc.},$$

an equation which obtains for each observed T and its corresponding A. If we take the mean of these several equations and note that the mean of the $(T - T_0)$s is necessarily zero, since T_0 is the mean of the Ts, we find for the average of the set,

$$\frac{1}{n}\Sigma A = A_0 + f''(A_0)\frac{1}{2n}\Sigma(T - T_0)^2 + \text{etc.}, \qquad (126)$$

where the last term of the expression is the required correction to reduce A_0 to the mean of the observed azimuths. For the numerical application of this formula we need to introduce a convenient expression for $f''(A_0)$, and there must also be a numerical factor such that the value of the term shall be given in seconds of arc when $T - T_0$ is expressed in minutes of time. This factor, combined with the coefficient $\frac{1}{2}$ which appears in the equation, is readily shown to be,

$$\left(\frac{60 \times 15}{206265}\right)^2 \times \frac{206265}{2} = [0.2930].$$

The differential coefficient, $f''(A_0)$, does not admit of an expression that is both simple and rigorous,* but, with entire accuracy at the pole and approximately for any star near the pole, we may write

$$f''(A_0) = \sin A_0,$$

* See p. 207 for the complete expression for $f''(A_0)$.

and combining these several expressions we find as the correction to the computed azimuth, A_0,

$$\Delta A_0 = +[0.2930] \sin A_0 \frac{1}{n} \Sigma (T - T_0)^2, \qquad (127)$$

where n is the number of Ts included in the mean, T_0, and the differences, $T - T_0$, are to be expressed in minutes of time. ΔA_0 must always be so applied as to bring the computed A_0 nearer to the meridian.

See § 85 for the extremely small effect of diurnal aberration upon azimuth determinations.

69. Example. — *Precise Azimuth.* — The example on p. 156 represents a determination of azimuth made with an engineer's transit, using the method of repetitions, four pointings in a set, and combining two sets in such a way as to eliminate the effect of lack of parallelism of the axes of the instrument, see § 53. The graduation errors are not here eliminated, and other sets with readings symmetrically distributed about the circle are required for this purpose.

70. Precise Latitudes. — *Zenith Telescope Method.*—In Fig. 12 let V represent any point on the meridian, S_1

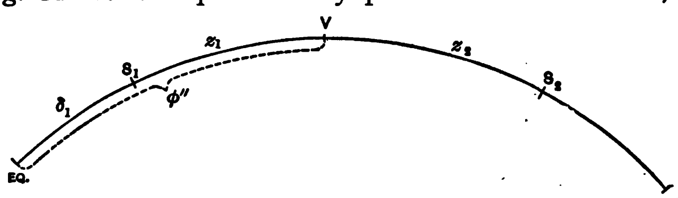


FIG. 12.—Zenith Telescope Latitudes.

and S_2 the points, on opposite sides of V, at which two stars, of declination δ_1 and δ_2 respectively, cross the meridian in their diurnal motion, and let z_1 and z_2 denote

PRECISE AZIMUTH DETERMINATION.

At Station M. Monday, May 1, 1899.
Instrument No. 386. Chronometer, S. Observer, C.
Chronometer $\Delta T = -2^m 39^s.7$. $4 \tan h . d = 10''.7$.

Object.	Circle n.	Chronometer.	Horizontal Circle. Ver. A.	Horizontal Circle. Ver. B.	Vert. Circle. Levels.	Angle.
		h. m. s.	° ′ ″	′ ″	° ′	° ′ ″
Mark....	*L*	12 25 ..	150 31 25	31 15	0 0	150 31 20
					W. E.	
Polaris..	*L*	12 27 28			8.5 15.0	
		30 59			12.7 11.0	
		33 28			−2.4	= −26
		36 24	117 21 40	21 35	41 50	117 21 38
	4	)128 19				)33 10 8
		12 32 4.8				8 17 32
Polaris..	*R*	12 40 7	117 22 0	21 30	41 43	117 21 45
		45 28			6.3 17.4	
		47 38			18.0 5.8	
		49 40			+0.55	= +6
Mark....	*R*	12 52 ..	150 10 0	9 40	0 0	150 9 50
	4	)182 53				)32 47 59
		12 45 43.2				8 12 0

REDUCTION.

ϕ	43 4 37	
δ	88 46 12.5	
a	1 21 28.0	
$\sec \phi$	0.13641	
$\cot \delta$	8.33180	
$\tan \phi$	9.97082	
g	8.46821	
k	8.30262	
$(T-T_0)^2$	20, 1, 2, 18	30, 0, 4, 16
$\frac{1}{4}\Sigma(T-T_0)^2$	1.009	1.097
$\sin A_0$	7.812n	7.682n
Const.	0.293n	0.293n
$\log \Delta A_0$	9.114	9.072
ΔA_0	+0″.13	+0″.12

	h. m. s.	h. m. s.
$T+\Delta T$	12 29 25.1	12 43 3.5
$T+\Delta T-a$	11 7 57.1	11 21 35.5
t	166° 59′ 16″	170° 23′ 52″
$\cos t$	9.98870n	9.99388n
$k \cos t$	8.29132n	8.29650n
$\sin t$	9.35249	9.22221
g	8.46821	8.46821
$F = -B_1$	9.99159	9.99149
$-\tan A_0$	7.81229	7.68191
A_0	179 37 41	179 43 28
ΔA_0	0	0
D	8 17 32	8 12 0
A'	187 55 13	187 55 28

The corrections ΔA_0 computed above are too small to be taken into account in observations of this character, but with a larger instrument or when the star is near elongation they become of sensible magnitude.

the arcs VS_1 and VS_2. Denoting by ϕ'' the declination of V, we have from the figure

$$z_1 = \phi'' - \delta_1, \qquad z_2 = \delta_2 - \phi'',$$

and by subtraction,

$$2\phi'' = (\delta_1 + \delta_2) + (z_1 - z_2). \tag{128}$$

Since V, by supposition, is any point of the meridian, we may now define it as the projection upon the meridian, of the point in which the vertical axis of a theodolite or other similar instrument meets the celestial sphere, and we may represent by b'' the zenith distance of V, reckoned positive when the zenith lies between V and the pole. Since the latitude is equal to the declination of the zenith, we shall have

$$2\phi = 2(\phi'' + b'') = (\delta_1 + \delta_2 + 2b'') + (z_1 - z_2). \tag{129}$$

In the practice of American government surveys all precise determinations of latitude are based upon this equation and are made with an instrument, the zenith telescope, especially designed for the micrometric measurement of small differences of zenith distance, the $z_1 - z_2$ of the equation. But Equation 129 may be applied with any instrument capable of measuring altitudes—theodolite, sextant, etc.—and in general it will furnish better results than other modes of using the instrument, since if the stars are so selected that z_1 differs but little from z_2, any constant errors which may be present in the instrumental work will be very nearly the same for the two stars, and will be approximately eliminated from the difference $z_1 - z_2$. We shall here

develop the zenith-telescope method with reference to its use with an engineer's transit provided with a gradienter and an altitude level, which latter may be its striding-level properly fastened to the alidade at right angles to the horizontal axis. With very small modifications the resulting formulæ will be applicable to the zenith telescope as usually constructed.

The first step in the application of the method is the selection of an observing programme, consisting of a number of pairs of stars whose right ascensions and declinations, for each pair, satisfy the conditions

$$\alpha_2 - \alpha_1 < 20^m, \qquad \delta_2 + \delta_1 - 2\phi < \pm G, \tag{130}$$

where G denotes the greatest angle that can be conveniently measured with the gradienter. Write upon the edge of a slip of paper the approximate value of 2ϕ, and turning to a suitable list of stars, e.g., the list of mean places given in the almanac, subtract each declination in turn from 2ϕ and seek within the given limits of right ascension a star whose declination differs but little from the difference thus obtained. If bright enough to be observed with the given instrument, any two stars thus related will constitute a latitude pair.

Having prepared such an observing list, before the first of these stars comes to the meridian let the instrument be carefully levelled and oriented and its telescope set to the approximate zenith distance of the star, $z_1 = \pm(\phi - \delta_1)$. When the star by its diurnal motion is brought into the field and passes behind the vertical thread, a pointing in altitude should be made upon it

PLATE V.

A Zenith Telescope as used at the International Latitude Stations Length of Telescope 52 inches. Approximate Cost $1600.

[*To face p.* 158.]

with the gradienter, and the readings of the altitude level and gradienter head recorded immediately after the pointing. Leaving the telescope firmly clamped in altitude, let it be now revolved 180° in azimuth without loosing the altitude clamp, and with the gradienter bring the line of sight to the zenith distance of the second star, $z_2 = \mp(\phi - \delta_2)$, and observe it precisely as before. If the level-bubble changes its position in the tube as the instrument is turned from the first to the second star, it should be brought back to its initial position by means of the levelling screws.

The readings of the level in the two positions determine the average value of b'', and if R_1 and R_2 represent the respective gradienter readings and k is the angle moved over by the line of sight when the gradienter is turned through one complete revolution, we shall have,

$$z_1 - z_2 = \pm k(R_1 - R_2). \qquad (131)$$

71. Minor Corrections.—Before introducing this value into the expression for 2ϕ we proceed to examine some matters that require further explanation, viz.:

Level Error.—The small term $2b''$ arises from a deviation of the vertical axis of the instrument from the true vertical Its amount and sign are to be determined from readings of an altitude level, as shown in § 42. Make this error small by turning the levelling screws, if necessary, so that the bubble readings shall be the same for the second star as for the first.

Refraction.—The effect of refraction upon the latitude observations is most readily determined by substituting,

in place of the true declinations of the stars, their apparent declinations as affected by the refraction. This displaces each star toward the zenith by the amount, (§ 23)

$$r=\frac{982''\,B}{456+t}\tan z; \tag{132}$$

and since for the southern star this displacement increases, while for the northern star it diminishes, the declination, we shall have as the sum of the apparent declinations,

$$\delta_1'+\delta_2'=\delta_1+\delta_2+\frac{982''\,.\,B}{456+t}(\tan z_1-\tan z_2),$$

which is equivalent to,

$$\delta_1'+\delta_2'=\delta_1+\delta_2+\left[\frac{982''\,.\,\sin 1^\circ}{\cos z_1\cos z_2}\cdot\frac{B}{456+t}\right](z_1-z_2)^\circ. \tag{133}$$

The following table gives the value of the bracketed coefficient in this equation, computed with the argument $z=\frac{1}{2}(z_1+z_2)$, for an average condition of the atmosphere, barometer 29.00 inches, temperature 50° Fahr. In all ordinary cases the correction for refraction may be found with sufficient accuracy by multiplying the tabular number, s, by the difference of the zenith distances of the two stars, expressed in degrees,

$$r=s(z_1-z_2)^\circ. \tag{134}$$

Since s is a positive number, the correction thus found will always have the same sign as the term z_1-z_2, measured with the gradienter.

REFRACTION COEFFICIENTS.

z	s		z	s	
0°	1.0″	0	50°	2.4″	6
10	1.0	1	55	3.0	9
20	1.1	2	60	3.9	16
30	1.3	4	65	5.5	29
40	1.7	7	70	8.4	61
50	2.4		75	14.5	

The use of the table is illustrated in the following short example taken from the data of § 73:

	°
z	50.9
z_1-z_2	5.6
s	2″.5
r	14.0

Reduction to the Meridian.—It is sometimes convenient or necessary to observe a star at some other instant than that of its meridian passage, and for this purpose the instrument may be turned out of the meridian, set at an azimuth that we will represent by a', and the observation made precisely as before. It is evident that this is equivalent to observing on the meridian a star whose meridian altitude is equal to the altitude of the given star at the moment of observation, and whose declination, therefore, differs from that of the latter star by the reduction to the meridian corresponding to the azimuth a', (Equation 55). In the reduction of the observation we have therefore to substitute in place of the star's true declination, δ, a corrected declination, δ'', given by the relation

$$\delta'' = \delta \pm f(a')^2, \quad f = [7.9407] \cos \phi \cos h_0 \sec \delta, \quad (135)$$

where a' is to be expressed in minutes of arc and the upper sign applies to a star between the zenith and pole, the lower sign to all other cases.

For the sake of increased precision it will frequently be advantageous to make several gradienter pointings upon a star in different azimuths, during the two or three minutes that precede and follow its culmination, and, having first oriented the instrument, to determine from readings of the horizontal circle the corresponding azimuths required in the reduction.

72. Errors of the Screw. — In Equation 131 it is tacitly assumed that the angle moved over by the line of sight when the gradienter is turned from one reading

to another is strictly proportional to the amount of turning of the screw. This is, however, an ideal condition seldom realized in fact, and if the capabilities of the instrument are to be fully utilized the errors of the gradienter must be investigated, and a set of corrections, C, determined, such that the angle moved through by the line of sight when the gradienter is turned from the reading R_1 to R_2 may be strictly proportional to the difference of the corrected readings, $R' = R_1 + C_1$, $R'' = R_2 + C_2$, i.e.,

$$z_1 - z_2 = \pm k(R' - R''). \qquad (137)$$

This calibration of the screw may be made as follows: Let some fixed vertical angle, e.g., the difference of elevation of two terrestrial points, be measured upon consecutive parts of the gradienter screw, from the beginning to the end of its run, so that, calling this angle v, we shall have,

$$\begin{aligned} v &= k[(R_1 + C_1) - (R_0 + C_0)], \\ v &= k[(R_2 + C_2) - (R_1 + C_1)], \\ v &= k[(R_3 + C_3) - (R_2 + C_2)], \qquad (137^*) \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ v &= k[(R_m + C_m) - (R_{m-1} + C_{m-1})]. \end{aligned}$$

The second reading of the screw in the first measurement of v must be the same as the first reading in the second measurement, etc., and to secure this the gradienter should not be touched after the second pointing, R_1, has been made, but the telescope should be unclamped, set back by hand, approximately, upon the first point

and the accurate pointing completed by means of the levelling screws.

From the mean of the preceding equations we obtain

$$\frac{v}{k}=\frac{R^m-R_0}{m}+\frac{C_m-C_0}{m}, \qquad (138)$$

which contains the three arbitrary quantities k, C_m, C_0, and is the only equation that these quantities are required to satisfy. We may therefore impose two additional relations among them, and, as convenient ones for the present purpose, we assume $C_m=C_0=c$, where c is a constant whose value we shall, for the present, leave undetermined. Representing by p the value of $\frac{v}{k}$ corresponding to these assumptions,

$$p=\frac{R_m-R_0}{m}, \qquad (139)$$

and introducing it into Equations 137, we find the following results:

$$\begin{aligned}
C_0 &= \qquad\qquad\qquad +c \\
C_1 &= (R_0+\ p)\ -R_1\ +c, \\
C_2 &= (R_0+2p)\ -R_2\ +c, \\
C_3 &= (R_0+3p)\ -R_3\ +c, \qquad (140) \\
&\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\
&\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\
C_m &= (R_0+mp)-R_m+c.
\end{aligned}$$

The corrections thus derived from the readings, R, may be plotted in a curve, from which values of C for all intermediate readings may be obtained. The particular value assigned to c will have no influence upon

the shape of this curve, but will determine its position with respect to the axis of x, and we may assign to c with advantage a value that will make the entire curve lie above the x-axis, i.e., one that will make all the values of C positive quantities.

The following example represents the record and reduction of a set of readings made for the investigation of the errors of the gradienter of an engineer's transit. The quantities in the column R are those directly observed; the column m gives the serial number corresponding to that used in the above analysis.

Thursday, June 7, 1900.
Gradienter of Instrument No. 386. Observer, *P*.

m	R	R_0+mp	$(R_0+mp)-R$	C
0	0.027	0.027	0.000	+31
1	2.037	2.025	−.012	19
2	4.045	4.023	−.022	9
3	6.047	6.022	−.025	6
4	8.048	8.020	−.028	3
5	10.049	10.018	−.031	0
6	12.047	12.016	−.031	0
7	14.045	14.014	−.031	0
8	16.035	16.013	−.022	9
9	18.022	18.011	−.011	20
10	20.009	20.009	0.000	+31

In the reduction of the above we use

$$p = \tfrac{1}{10}(20.009 - 0.027) = 1.9982.$$

The last column, expressed in thousandths of a revolution, is obtained by adding to the numbers of the preceding column the assumed constant, $c = +0.031$. A considerable number of such determinations should be made and the mean of the several results adopted as definitive corrections to the gradienter readings. Similar

corrections must always be applied where a high degree of precision is required in the use of a gradienter or other similar micrometer, e.g., the eyepiece micrometer of a zenith telescope or transit, and particular care should be given to them in determinations of k, the value of one revolution of the gradienter screw.

In a similar manner the gradienter should be examined for periodic errors, i.e., errors peculiar to a particular part of a turn and which repeat themselves whenever the same part of the head, as the o, comes under the index, regardless of the number of whole revolutions at which the screw stands.

73. Gradienter Latitudes. Example. — We may now write the equation for zenith-telescope latitudes in the form,

$$2\phi = \delta_1' + \delta_2' + 2b'' \pm [k(R' - R'') + r], \qquad (141)$$

through which a value of the latitude may be derived from each pair of stars observed, if k is known. This *value of a revolution of the screw*, k, may be determined by measuring with the gradienter a known angle, such as the difference of declination of two stars, or it may be treated as an unknown quantity whose value is to be derived from the latitude observations themselves. In the latter case at least two pairs of stars, preferably ten to twenty pairs, must be observed for the determination of the two unknowns, ϕ and k, and these should be so selected that in one pair the sum of the declinations is greater than 2ϕ and in the other pair is less than 2ϕ.

The following example represents the observation

and reduction of a single pair of stars made with the instrument shown in Plate I, whose errors are investigated in § 72. The gradienter readings as directly observed are given in the column marked R, and in the following column there are given the corrections to these readings as interpolated from the table at p. 164. The instrument having been oriented by the method of § 32, the readings of the horizontal circle, in the column H.C., furnish immediately the azimuths, a', required for computation of the reductions to the meridian, which are here represented by the letter M. The stars' meridian altitudes, h_0, that are also needed for the computation of these reductions, may be obtained with sufficient accuracy from the declinations and the known approximate latitude of the place, 43°. The value of a revolution of the gradienter, k, was known to be about 20′ 30″, and this value together with the observed difference of the gradienter readings determines $z_1 - z_2$ with sufficient precision to permit the refraction correction to be interpolated from the table at p. 160. The level correction, $2b'' = -7''$, is negative since the level readings show that the vertical axis of the instrument pointed north of the zenith, i.e., in too great a latitude.

The declinations of the stars are taken from the American Ephemeris, but in the case of Polaris, which was observed at its transit over the lower half of the meridian, *sub polo*, the almanac declination is subtracted from 180° in order to obtain the distance of the star from the upper half of the equator, which is the quantity used in the analysis and required in the reduction.

Monday, May 20, 1901.
At Azimuth Stake. Instrument No. 389. Observer, C.

Star.	Level. N S.	R	Corr.	H. C.	δ	M	Remarks.
				° ′	° ′ ″	″	
Polaris, S.P.	6.2 9.2	0.302	+29	179 55	91 13 18	−6	Level, $d=3''$
α Virginis	8.0 7.5	16.704	+13	358 21	−10 38 57	−51	
	−1.25						

Reduction to Meridian.

Star	Polaris	α Virginis
a'	$5'$	$99'$
$\cos\phi$	9.863	9.863
$\cos h_0$	9.872	9.906
$\sec\delta$	1.671	0.008
Const.	7.941	7.941
$(a')^2$	1.398	3.992
$\log M$	0.745	1.710
M	−5.6	−51.2

$\delta_1' + \delta_2'$	80° 33′ 24″
$2b''$	−7
Ref'n	+14
$R' - R''$	16.386 rev.

$$2\phi = 80^\circ\ 33'\ 31'' + 16.386k$$

Each observed pair of stars furnishes an equation similar to the above, involving ϕ and k as unknown quantities, for which definitive values are to be obtained from a solution of all the available equations. For illustration we select a single additional pair of stars and its resulting equation, viz.,

$$2\phi = 91^\circ\ 9'\ 52'' - 14.671k,$$

and combining it with the one derived above we obtain,

$$k = 1229''.4, \qquad \phi = 43^\circ\ 4'\ 38''.$$

This value of ϕ agrees within 1″ with the known latitude of the place of observation and represents about the limit of accuracy attainable with an engineer's transit.

With the zenith telescope, used in essentially the same manner as above, a precision of about 0″.1 is attained. See Appendix 7, Report of the U. S. Coast and Geodetic Survey for the Year 1897–98, for an exposition of the methods employed with such an instrument.

CHAPTER IX.

THE TRANSIT INSTRUMENT.

74. **General Principles.** — *Adjustments of the Instrument.*—If the celestial meridian were a visible line drawn across the heavens, the local sidereal time corresponding to this meridian might be determined by observing the chronometer time, T, at which a star of known right ascension, α, crossed this line. We should then have for the correction of the timepiece employed,

$$\Delta T = \alpha - T.$$

The transit instrument, different forms of which are shown in the Frontispiece and in Plate VI, is a substitute for the visible meridian above supposed. Its essential parts are illustrated by the telescope and standards of a large theodolite firmly mounted, with the horizontal axis of rotation perpendicular to the plane of the meridian, i.e., east and west, and level. The telescope is usually provided with several vertical threads (an odd number of them), each of which, as seen by the observer, is projected against the sky as a background, and each of which, when the telescope is turned about the rotation axis, traces upon the sky, by virtue of this rotation, a circle whose plane is perpendicular to the axis. Also, one or more horizontal threads are usually introduced to mark the middle points of the transit threads.

PLATE VI.

A Straight Transit Instrument. Length of Telescope 30 inches.
Approximate Cost $1000.

[*To face p.* 168.]

A transit instrument is said to be perfectly adjusted when the circle thus traced upon the sky by its middle vertical thread coincides with the local meridian, and for such an instrument it is evident that the time of a star's transit over this thread may be substituted for the time of its transit over the visible meridian above supposed, and the chronometer correction, ΔT, will then be furnished by the equation printed above. But in general it cannot be assumed that these adjustments are perfect, and we must consider them as so many possible sources of error whose effects must be in some way eliminated from the results of observation.

Optical Adjustments.—We assume that great care has been given to the optical adjustment of the instrument, so that both the transit threads and the star are sharply defined and distinctly seen. For this purpose the eyepiece should first be so set that the threads appear black and distinct, and threads and eyepiece should then be moved in or out together until a star, preferably a double star, presents a clear image without trace of fuzziness, projecting rays, or stray light. This last adjustment may be a little more accurately made by covering the upper half of the telescope objective with cardboard or paper and making an accurate pointing of the horizontal thread upon a circumpolar star near culmination. Having made a satisfactory pointing, quickly shift the card so as to cover the lower half of the objective and leave free the upper part, when, if the threads are not properly adjusted with respect to the objective, there will be a slight vertical displacement of the star

with respect to the thread, and this must be corrected by further adjustment.

Verticality of Threads.—To make the threads perpendicular to the rotation axis, point the telescope at a terrestrial mark, and turning the telescope in altitude with the slow-motion screw, note whether the mark in its apparent motion up and down the field of view runs exactly along the thread. Any outstanding error in this adjustment may be removed by slightly rotating in its own plane the collar which carries the threads; but a small error here may be rendered harmless by always pointing the telescope, at the times of observation, so that the stars cross the same part of the field, e.g., between the parallel horizontal threads.

The principal errors of adjustment that remain to be considered in connection with the use of a transit instrument are three in number, viz.: The *azimuth error*, a, is the angular amount by which the rotation axis deviates to the south of due west. The *level error*, b, is the angle of elevation of the rotation axis above the western horizon. The *collimation error*, c, is the amount by which the angle between the line of sight and the west half of the rotation axis exceeds 90°. The line of sight as here used means the imaginary line passed through the optical centre of the objective and the middle transit thread, or through the mean of a group of transit threads.

75. Theory of the Instrument.—To determine the relation of these several instrumental errors to the time, T, at which a star will pass behind a given transit thread

we have recourse to Fig. 13, which represents a projection of the celestial sphere upon the plane of the horizon. Z is the projection of the zenith, P of the pole, H of the point in which the rotation axis, produced toward the west, intersects the celestial sphere, and S is the projection of a star observed at the moment of its transit over

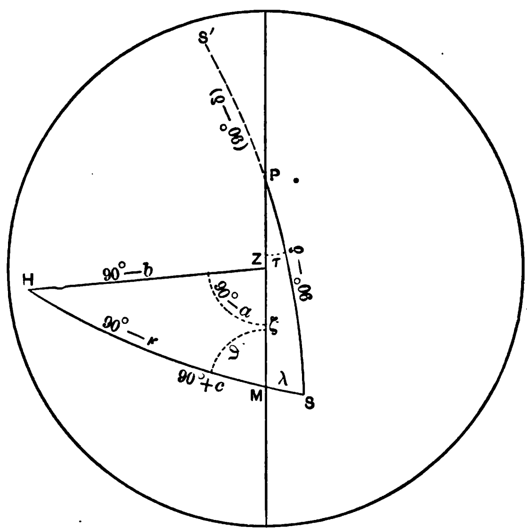


FIG. 13.—The Transit Instrument.

a thread whose angular distance from H is measured by the arc $90° + c$. From the definitions given above, c represents the collimation of the particular thread in question, and similarly b and a, in the figure, are the level and azimuth errors above defined. The symbol τ of the figure represents the hour angle of the star, reckoned toward the east, δ is the star's declination, $90° - \kappa$ is the arc HM, and λ is the distance of the star from the meridian measured along HS. This latter arc

must not be confounded with the diurnal path of the star; the one is an arc of a great circle defined by the points H and S, while the other is a small circle having its pole at P. Note that in all cases the symbols here defined represent the actual magnitudes of the arcs and angles on the sphere, and not of their projections on the plane of the horizon.

From the spherical triangle PMS we obtain the relation,

$$\cos \delta \sin \tau = \sin \lambda \sin \vartheta, \tag{143}$$

and from the triangle ZHM we find,

$$\sin \kappa = \sin b \cos \zeta + \cos b \sin \zeta \sin a. \tag{144}$$

These equations may be greatly simplified by substituting arcs in place of sines whenever the quantities a and b are so small that their cubes and higher powers may be neglected, and we shall therefore assume that we have to deal with an approximately adjusted instrument, in which neither of these quantities much exceeds $10'$. On this supposition the point H is so nearly the pole of the meridian, PZM, that we may put $\sin \vartheta = 1$ and $\zeta = \phi - \delta$, where ϕ denotes the latitude of the place of observation, and our equations now take the form,

$$\left.\begin{aligned} \tau &= \lambda \sec \delta, \\ \kappa &= b \cos (\phi - \delta) + a \sin (\phi - \delta). \end{aligned}\right\} \tag{145}$$

From the figure we have the relation,

$$90° + c = 90° - \kappa + \lambda,$$

and eliminating λ and κ between these equations we find,

$$\tau = \sin(\phi - \delta) \sec\delta . a + \cos(\phi - \delta) \sec\delta . b + \sec\delta . c. \quad (146)$$

Since τ is an east hour angle, we have also, in terms of the observed time, the chronometer correction, and the star's right ascension,

$$T + \varDelta T = a - \tau, \quad (147)$$

from which we obtain, by the elimination of τ, Mayer's equation of the transit instrument,

$$a - T = \varDelta T + \sin(\phi - \delta) \sec\delta . a \\ + \cos(\phi - \delta) \sec\delta . b + \sec\delta . c, \quad (148)$$

or, as it is usually written,

$$a - T = \varDelta T + Aa + Bb + Cc, \quad (149)$$

where the capital letters are introduced as abbreviations for the coefficients given above, i.e.,

$$A = \sin(\phi - \delta) \sec\delta, \ B = \cos(\phi - \delta) \sec\delta, \ C = \sec\delta. \quad (150)$$

Since a, T, and $\varDelta T$ are expressed in time (hours, minutes, and seconds), it is customary in connection with this equation to express a, b, and c in seconds of time.

76. Discussion of Mayer's Equation.—The coefficients A, B, C are called *transit factors*, and when many observations are to be made in the same latitude, ϕ, as at an observatory, it is customary to tabulate their values with the declination as argument, and to interpolate from these tables the values of the factors corresponding to the particular stars observed. In the U. S. Coast and Geodetic Survey Report for the year 1880 there may be

found extensive tables of this kind for different latitudes covering the whole extent of the United States.

In the use of such tables the following distinction must be carefully observed: Every star whose distance from the pole is less than the latitude remains continuously above the horizon throughout the twenty-four hours, and during this period crosses the meridian twice, once above the pole, e.g., between the pole and zenith, and once below the pole, e.g., between the pole and the northern horizon. The latter transit is usually designated *sub polo*, and from Fig. 13, where S' represents the star S near its transit *sub polo*, it may be seen that its coordinates at this transit will be obtained by substituting in place of the α and $90° - \delta$, corresponding to S, $12^h + \alpha$ and $-(90° - \delta)$. When these values are introduced into Mayer's equation it becomes, for stars observed *sub polo*,

$$12^h + \alpha - T = \Delta T + A'a + B'b + C'c, \qquad (151)$$

where the new transit factors have the following values:

$$A' = \sin(\phi + \delta) \sec \delta, \quad B' = \cos(\phi + \delta) \sec \delta, \quad C' = -\sec \delta. \qquad (152)$$

As an exercise in analysis the student may show that the transit factors for a star above and below the pole are connected by the relations,

$$A + A' = 2 \sin \phi, \quad B + B' = 2 \cos \phi, \quad C + C' = 0. \qquad (153)$$

Use these equations to derive A', B', C' from the tabulated values of A, B, and C.

From a consideration of the trigonometric functions

that enter into the transit factors the algebraic signs of these factors are found to be as follows for places in the northern hemisphere:

Factor.	A	B	C
South of Zenith....	+	+	+ −
Zenith to Pole.....	−	+	+ −
Below Pole	+	−	− +

Note that in every case the transit factors for a given star have opposite signs above and below the pole, and compare with this statement the fact that stars on opposite sides of the pole move in opposite directions, east to west above pole and west to east below pole.

Query.—The above relations of sign are for a place in north latitude. How must they be changed to adapt them to a place south of the terrestrial equator?

In explanation of the double set of signs given above for C, we recall what was shown in § 50, that a reversal of the instrument changes the sign of the collimation constant, c; i.e., $90°-c$ is substituted for the $90°+c$ of Fig. 13, by lifting the axis out of the wyes and replacing it, turned end for end. It is customary to ignore this change of sign in c, and to represent its effect in Mayer's equation by changing the algebraic sign of C when the instrument is reversed; e.g.,

For Circle W.........$C(+c)=(+C)c$
For Circle E.........$C(-c)=(-C)c$

The collimation constant, c, may be either positive or negative, depending upon the adjustment of the instrument; but it retains the same sign in both positions of the

circle, while the collimation factor, C, is positive (above pole) when the circle end of the axis points west, negative when it points east.

77. **Choice of Stars.** — In the right-hand member of Mayer's equation, as printed on page 173, there are involved four unknown quantities, ΔT, a, b, and c, one of which, b, the inclination of the axis to the plane of the horizon, is always to be determined by some mechanical method, e.g., the use of a spirit-level. The collimation constant, c, may also be determined mechanically (see § 84), but for the present we shall assume that this has not been done and that the instrumental constants a and c, as well as the clock correction ΔT, are to be determined from observations of stars. Since there are three quantities to be thus determined, there must be at least three observations, and it is practically convenient to make four the minimum number instead of three; observing two stars Circle E. and two Circle W. for the sake of a good determination of the collimation, c, through the reversal of the instrument. The stars thus chosen should not all lie on the same side of the zenith, but should be distributed on both sides, so as to make the sum of their azimuth factors as small as possible. When $\Sigma A = 0$, the effect of the azimuth error, a, is completely eliminated, and a nearly complete elimination may usually be obtained by care in the selection of stars. In the example of § 78 this condition is approximately satisfied by the four stars marked a', b', d', e', and the student after tracing through the reduction there given, should note that if the azimuth star, 1 H. Draco., were

dropped and the azimuth error entirely ignored, the resulting value of ΔT would be substantially the same as is obtained when the azimuth error is taken into account. In this case, therefore, an accurate determination of a is of little consequence.

78. **Example.**—*Ordinary Determination of Time.*—The following example, taken from the time service of the Washburn Observatory, $\phi = 43° 4' 37''$, illustrates the record and reduction of a set of transit observations. In addition to the date and the measured inclination, b, of the horizontal axis, given in the column of Constants for the two positions of the instrument, Circle W. and Circle E., the observed data are contained in the three columns marked, at the foot, with Roman numerals, I, II, III. The observed times of transit given in III are each the mean of the observed times of transit of the given star over 15 threads, and in the reduction the collimation constant, c, is assumed to refer to the mean of these threads instead of to the middle thread. Note that this particular convention with regard to c can be adopted only when each star is observed over precisely the same set of threads as every other star. The failure to observe a single star at its transit over one of the threads will require either the rejection of the transits of other stars observed at this thread, or a determination of thread intervals and a "reduction to the mean thread" for which reference may be made to Appendix 7, U. S. Coast and Geodetic Survey, Annual Report for 1897–98.

The remaining columns are marked with Arabic numerals, showing the order in which they are reached in

the computation. Of these columns 1 and 2 are obtained from the almanac (in this case the *Berliner Astronomisches Jahrbuch*, plus the corrections given in *Astronomische Nachrichten*, No. 3508). The declinations are taken to the nearest minute only, while the right ascensions are accurately interpolated for the instant of the star's transit over the local meridian, i.e., 0.3 day after their transit over the meridian for which the almanac is constructed. The third star, being observed *sub polo* (and before midnight), was observed half a day before its transit over the local upper meridian, and its right ascension is therefore interpolated for an instant 0.2 day *before* its transit over the Berlin meridian.

The transit factors contained in columns 4, 5, and 6 were interpolated from tables of such factors, and the products contained in columns 7 and 8 were next filled in by the use of Crelle's multiplication tables. It may be noted that the effect of diurnal aberration shown in column 7 has already been found (§ 27) to be $-0^s.021 \cos\phi \sec\delta$, which, for the given latitude, is equal to $-0^s.015\,C$, and the collimation factor C was employed in computing the correction. These corrections were next added mentally to the numbers contained in III, and the resulting times subtracted from the right ascensions in 1, thus giving the absolute terms of the equations numbered 9. The first members of these equations, 3, 4, 6, are obviously derived from Mayer's equation.

We have now five equations involving only three unknown quantities and presenting, therefore, a case for the application of the Method of Least Squares. A rigorous solution by that method furnishes the following values of the quantities sought:

$$\varDelta T = +2^m 57^s.010, \qquad a = +0^s.858, \qquad c = +0^s.966.$$

But such a solution is rather laborious, and a simple method of obtaining approximately accurate results is indicated under the heading Solution, where the symbols at the left indicate the manner in which the successive equations are derived. Equation k' is derived from i' by dividing through by the coefficient of c, and l' is similarly derived from h', using the coefficient of $\varDelta T$, as divisor and substituting in place of c its value given by k'. Equation m' is obtained from c' by substituting in place of $\varDelta T$ and c their values as given in k' and l'.

The value of a furnished by this equation when substituted in k' and l' gives definitive values of ΔT and c, all of which are entered in the column of Constants.

By means of these values of a and c, columns 12 and 13 are filled up and the sum of the corrections contained in columns 7, 8, 12, 13, is entered in 14 and added to the corresponding numbers in III, thus furnishing the corrected times contained in 15. Only the seconds are entered here, since the minutes remain unchanged. The individual values of the clock correction contained in 16 are now obtained by subtracting 15 from 1, and their agreement, one with another, is a check upon the accuracy of the entire work, both observations and computations. For the sake of this check it is better to proceed as is here done than to rely upon the value of ΔT furnished by the solution of the equations. The numerical work here shown is greatly facilitated by the use of a slide-rule or an extended multiplication table such as that of Crelle.

It may readily be seen from the course of the above solution that the collimation, c, is obtained from the four observations marked a', b', d', e', while the azimuth, a, is furnished by the third observation. A star near the pole, like 1 H. Draco., is introduced into the observing programme solely to determine a, and with reference to this use it is called an *azimuth star*, while the others are known as *clock stars*, since it is they that determine the value of ΔT. As there is always a possibility of disturbing the azimuth, i.e., changing a in the act of reversing the instrument, there should, in all strictness, be two values of a determined, one for Circle W. as well as the one above found from the observation of a star Circle E.; but in the present case it may readily be seen that there was no such disturbance, since the value of a for Circle E. brings into perfect agreement the values of ΔT furnished by the two stars observed Circle W., although their azimuth factors are widely different.

Whenever necessary, introduce into the solution two azimuths, one for each position of the instrument, as

TIME DETERMINATION WITH TRANSIT.

Sunday, Sept. 10, 1893. Observer, A. C.

Star.	Circle.	T.	Di. Ab.	Aa.	Bb.	Cc.	Sum.	Corr'd T.	a.	ΔT.	Constants.
		h. m. s.	s.	s.	s.	s.	s.	s.	h. m. s.	m. s.	s.
a' α Equulei	W.	21 7 32.83	−.02	+ .53	−.02	+ .96	+ 1.45	34.28	21 10 31.30	+2 57.02	W. b = −0.020
b' α Cephei	W.	13 5.46	−.03	− .60	−.04	+2.06	+ 1.39	6.85	16 3.87	57.02	E. b = −0.149
c' 1 H. Draco. S.P.	E.	18 39.80	+.10	+4.90	+.59	+6.74	+12.33	52.13	21 49.15	57.02	a = +0.855
d' g Cygni	E.	22 37.07	−.02	− .06	−.21	−1.39	− 1.68	35.39	25 32.44	57.05	c = +0.964
e' 74 Cygni	E.	29 46.49	−.02	+ .06	−.19	−1.25	− 1.40	45.09	32 42.07	56.98	ΔT = +2 57.016
I	II	III	7	12	8	13	14	15	1	16	11

	δ.	B.	A. C.	
a'	+ 4° 48′	+0.79	$\Delta T + 0.62a + 1.00c =$	$+2^m 58^s.51$
b'	62 8	+2.02	$\Delta T - 0.70a + 2.14c =$	2 58.48
c'	81 48	−3.99	$\Delta T + 5.73a + 6.99c =$	3 8.66
d'	46 4	+1.44	$\Delta T - 0.07a - 1.44c =$	2 55.60
e'	39 55	+1.30	$\Delta T + 0.07a - 1.30c =$	2 55.79
	2	5	3 4 6	9

Solution.

$f' = a' + b'$ 5 $56.99 = 2\Delta T - 0.08a + 3.14c$

$g' = d' + e'$ 5 $51.39 = 2\Delta T + 0.00a - 2.74c$

$h' = f' + g'$ 11 $48.38 = 4\Delta T - 0.08a + 0.40c$

$i' = f' - g'$ $5.60 = 0\Delta T - 0.08a + 5.88c$

k' $c = +0.952 + 0.014a$

l' $\Delta T = +2\ 57.000 + 0.019a$

m' $+5.00 = +5.85a$

10

unknown quantities. It is not necessary to introduce two collimations.

79. Methods of Observation.—A clock or chronometer is an indispensable auxiliary to a transit instrument, and an observation with the latter consists in determining, as accurately as may be, the chronometer time at which a particular star transits over one or more of the threads. In the best astronomical practice a recording machine, called a chronograph, is used in this connection, but we shall here suppose the observer not to be provided with a chronograph and constrained, therefore, to use the older method of observing by *eye and ear*. In this method the observer picks up the beat of the chronometer, i.e., counts mentally the tick corresponding to each successive second, 1, 2, 3, 4, etc., and while thus counting looks into the telescope and watches the progress of the star across the field of view, noting its position at the instant of each counted beat. If, by any chance, the star should appear exactly behind a thread at the instant when the counted beat was 26, the time of transit over this thread would be recorded 26.0 seconds, and the corresponding hour and minute subsequently determined by looking at the face of the chronometer. It will usually happen, however, that the star passes behind the thread between two chronometer beats instead of simultaneously with one of them, somewhat as shown in Fig. 14, where there is indicated the position of the star with respect to the thread at 26^s and at 27^s, as noted

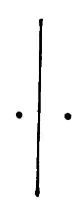


FIG. 14.—Transits by Eye and Ear.

and temporarily remembered by the observer. From the manner in which the thread divides the space between the two star images it is evident that the actual transit over the thread occurred at 26.4^s, and it should be so recorded. The fraction of a second depends upon the observer's estimation (an estimate of space seen in the telescope and not time as counted by the ear), and a skilled observer should be able to follow a star in its progress across the field of view, observing and recording to the nearest tenth of a second the times of transit over as many threads as may be desired, without taking the eye from the telescope during the process. He should, while watching the star, give no heed to the hour and minute, but concentrate attention upon the seconds and fractions of a second, until the transit over the last thread has been recorded; then, still counting seconds, let him look back at the face of the chronometer and note if the time there shown by the seconds hand agrees with his count. This is called *checking the beat,* and if it checks properly, the minute and hour corresponding to the last observation should be written down as a part of the record.

80. Precision of the Results. — By the method above outlined a skilled observer may, from the mean of several threads, determine the time of a star's transit within very small limits of error; e.g., there is found for the probable error of a transit of a single star over the mean of from 10 to 15 threads, some $0^s.02$ or $0^s.03$. But this apparent precision is in some degree fallacious, for most observers possess individual peculiarities, called *personal equation,*

by which they tend to observe all stars either too soon or too late, by a nearly constant amount, and the probable error of a transit based upon the agreement of individual results, one with another, furnishes no indication of the presence or magnitude of this constant personal error.

Closely related to the precision attainable in estimating the times of transit of a star over the threads of an instrument, is the degree of accordance to be expected among the values of ΔT furnished by the several stars composing a set, such as that of the illustrative example of § 78. The range of values there exhibited, while smaller than is to be expected from a beginner, may be regarded as fairly typical of the results to be obtained by an experienced observer provided with a good instrument. See in this connection the example of § 82, where the results show an even closer but by no means abnormal agreement.

81. Personal Equation. — The personal equation, although a real and oftentimes a considerable source of error, is, however, of small consequence save where the observations of different persons are to be combined, one with another, as in a determination of longitude. In such cases, however, the problem of personal equation must be met and seriously dealt with, and various devices have been employed for this purpose; e.g.: (1) An exchange of observers at the middle of the work in question, so that its first half may be affected with the personal error in one direction and the second half in the opposite direction, thus eliminating this influence from the mean.

(2) The determination of the exact amount of the personal equation for each observer, by means of so-called personal-equation machines, is sometimes attempted; but at present the best device for the elimination of personal equation seems to be: (3) The Repsold Transit Micrometer, an apparatus in whose use the methods of observing above set forth, § 79, are completely abandoned, and as a substitute for them the observer, while looking into the telescope, seeks to keep the image of a star, as it moves across the field, constantly covered by a micrometer thread, which he manipulates with his fingers and which is so connected with a chronograph as to give an automatic record of the star transits. The experience of the Prussian Geodetic Institute indicates that in this mode of observing, personal differences between observers are nearly annihilated.

82. Reversal of the Instrument upon Each Star.—A method of using a transit instrument introduced into general practice in connection with the transit micrometer, but which may be equally well applied with the ordinary chronographic or eye-and-ear methods, consists in noting the time of transit of a star over a group of threads placed at some little distance from the centre of the field, then, after quickly reversing the instrument, to observe the same star again on the same threads in their new position. It is obvious that the effect of collimation is thus completely eliminated from the mean of the observations on each thread and therefore from the general mean of the observed times. This elimination, while an important advantage of this mode of ob-

serving, is far from being the only one, and a considerable number of sources of error which have not been considered above, but which are dealt with at length in the larger treatises, such as Chauvenet, *Spherical and Practical Astronomy*, are equally eliminated by the reversal; e.g., inequality of pivots, flexure, thread intervals, and the disturbance of the spirit-level incident to reversing it upon the axis. When the telescope is reversed upon every star a hanging level may be allowed to remain upon the axis without ever being reversed, since the level readings in the two positions of the axis then give its mean inclination, which is the datum required for the reduction of the star transits.

Whenever it can be employed the method of reversal upon every star is to be preferred to the older method illustrated in the preceding example, but it requires special facilities for quick reversal of the instrument without disturbing its azimuth, and these are not always present.

The following is an example of the record and reduction of such a series of observations, made with the same instrument and arranged in nearly the same manner as the example on p. 180. Each star was observed on five threads in each position of the instrument, and a value of the level constant, b, was determined for each star from readings of the hanging level, taken immediately before or after the observed transits in each position of the circle, the level remaining unreversed upon the axis during the entire set of observations.

TIME DETERMINATION WITH TRANSIT.

Wednesday, Oct. 5, 1898. Observer, C.

Circle W. and E. for each star.

	Star.	b	T	Di. Ab.	Bb	Aa	Corr'd T.	α	ΔT
		s.	h. m. s.	s.	s.	s.	s.	h. m. s.	s.
a'	α Lyræ	−0.23	18 34 24.22	−.02	−.29	+.06	23.97	18 33 31.07	−52.90
b'	110 Herculis	− .25	42 11.89	−.02	−.25	+.25	11.87	41 19.00	−52.87
c'	β Lyræ	− .22	47 14.17	−.02	−.26	+.12	14.01	46 21.14	−52.87
d'	o Draconis	− .24	50 36.30	−.03	−.45	−.34	35.48	49 42.61	−52.87
	I	II	III	5	6	8	9	1	−52.878

	δ	B	A	Solution.
a'	38° 42′	1.28	+0.10	$\Delta T+0.41a=-52.62$
b'	20 27	0.98	+0.41	$\Delta T-0.55a=-53.21$
c'	33 15	1.18	+0.20	$+0.96a=+\ 0.59$
d'	59 16	1.88	−0.55	$a=+\ 0.61$
	2	4	3	7

A least-square solution gives $a=+0^s.611$, $\Delta T=-52^s.879$.

83. Determination of Azimuth with a Transit Instrument.—Let the line of sight of a transit instrument be supposed directed accurately upon some terrestrial mark, and the telescope then turned up to the sky and the time of a star's transit over the line of sight observed. From this observed time the hour angle of the star may be derived, and this hour angle, in connection with the known declination and latitude, will determine the star's azimuth at the instant of observation. If there are no instrumental errors present, this computed azimuth will be the true azimuth of the terrestrial mark at which the line of sight was originally directed.

This simple method of determining azimuth requires some modifications on account of instrumental errors, but when these are duly taken into account and a proper selection of stars and mark is made, the method ranks as the best of all known ones for azimuth determination. The star to be observed should be very near the pole, usually Polaris, and if the chronometer correction, ΔT, is accurately known, the observation may be made at any convenient time, e.g., the time at which the star stands directly above a mark already established. If the chronometer correction is not well determined, the observation should be made when the star is near elongation, since the effect upon the computed azimuth of an error in the assumed ΔT is then a minimum. But this latter procedure requires the establishment of a special mark whose azimuth shall be very approximately equal to that of the star when at elongation, and it will often be more convenient to determine the time with

the required accuracy, e.g., one tenth of a second, and thus obtain more freedom in the choice of a mark.

A transit instrument of the better class is usually provided with an eyepiece micrometer, i.e., one or more threads parallel to the fixed transit threads, but capable of being moved to and fro in the field of view by a screw whose axis is parallel to the rotation axis of the instrument. This screw is provided with a graduated head whose readings indicate the successive positions of the thread and measure the amount of its motion between consecutive pointings upon the star and mark. When such a micrometer is present, transits of the star may be observed over its threads as long as the star remains within the field of view, and many comparisons between star and mark may be substituted for the single one above supposed. The instrument should be reversed at least once during these observations, and the inclination of its axis, b, must be carefully determined since, as will appear later, the level error has an important effect upon the azimuth.

84. Theory of the Method. — To derive from the micrometer readings upon star and mark the difference of their respective azimuths we have recourse to Fig. 15, which represents a projection of the celestial sphere upon the plane of the horizon. S and M represent respectively the star and the mark, Z is the zenith, and the spherical angle SZM is the required difference of azimuth. Let H be the point in which the rotation axis of the instrument, produced toward the west, meets the celestial sphere, and the arcs $90° - b$, $90° + c$, will then have the

same significance as in Fig. 13. The spherical angles HZS and HZM are represented by the symbols $90°+w$

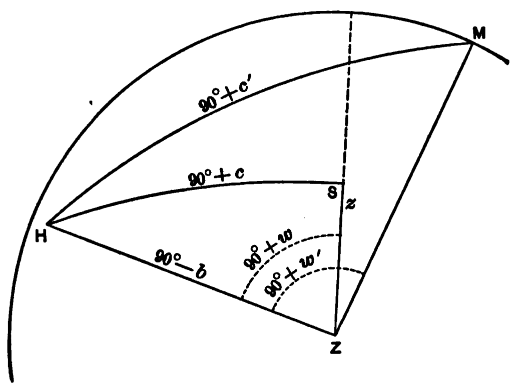


Fig. 15.—Azimuth with a Transit Instrument.

and $90°+w'$, and the zenith distance of the star, ZS, by z.

From the triangle HZS we obtain,

$$\cos(90°+c) = \cos(90°-b)\cos z + \sin(90°-b)\sin z\cos(90°+w), \quad (154)$$

which, when b and c do not much exceed $10'$, is equivalent to,

$$c + b\cos z = w\sin z. \quad (155)$$

In this equation we substitute $90°-h$ in place of z and put $\sec h = 1+\sigma$, and it becomes,

$$w = c + c\sigma + b\tan h. \quad (156)$$

From the triangle ZHM we find in a similar manner for the mark,

$$w' = c' + c'\sigma' + b\tan h'. \quad (157)$$

When the mark, M, is in or very near the horizon, as it should be, the last two terms of Equation 157 vanish and we obtain by subtracting it from Equation 156,

$$A - A' = w - w' = c - c' + c\sigma + b \tan h. \qquad (158)$$

Let k represent the angular equivalent (value) of one revolution of the micrometer screw, R the reading of the screw-head corresponding to any position of the movable thread, and R_0 the particular reading at which the angle between the rotation axis of the instrument and the line of sight defined by the thread equals 90°, i.e., R_0 is the reading corresponding to $c = 0$. For any other position of the threads corresponding to the reading R we shall have

$$c = \pm k(R - R_0), \qquad (159)$$

where the ambiguous sign depends upon the position of the instrument, whether Circle W. or Circle E. For any given instrument it is well to determine, by trial, once for all, in which of these positions the readings of the micrometer head continuously diminish as the micrometer thread is made to follow the diurnal motion of a star near upper culmination, and, with reference to the sign in Equation 159, designate this as the positive, the other as the negative, position of the instrument.

Corresponding to the positive and negative positions, respectively, let R_1 and R_2 be readings of the micrometer head when the micrometer thread is pointed upon the same fixed object, e.g., the mark whose azimuth is to

be determined; we shall then have as the distance of this object from the line of no collimation,

$$\begin{aligned} &\text{Positive Position,} \quad s = +k(R_1 - R_0), \\ &\text{Negative Position,} \quad s = -k(R_2 - R_0), \end{aligned} \tag{160}$$

from which we readily obtain,

$$\begin{aligned} 2s &= +k(R_1 - R_2), \\ 2R_0 &= R_1 + R_2. \end{aligned} \tag{161}$$

The first of these equations determines the distance, s, of the terrestrial mark from the collimation axis of the instrument, and it should be used to make the distance of the azimuth mark small, by properly placing the instrument, whenever an azimuth determination is to be made. The second equation determines R_0, and through R_0 the collimation corresponding to any position of the micrometer thread may be found; e.g., let R denote the reading of the micrometer when the movable thread is placed in apparent coincidence with any fixed thread of the transit reticule, then will the collimation of this thread be given by Equation 159. This method of determining collimation may be employed in connection with time determinations, as indicated in § 77.

To apply these equations to the reduction of a set of azimuth observations we let S represent the mean of several micrometer readings made in quick succession upon the star, and similarly we represent by M the mean of several readings to the mark. Introducing these quantities into Equation 159, we find for the star and mark, respectively,

$$c = \pm k(S - R_0), \quad c' = \pm k(M - R_0), \tag{162}$$

and substituting these values in Equation 158, we obtain

$$A-A'=\pm k\{(S-M)+\ \sigma\ (S-R_0)\}+b\tan h. \quad (163)$$

This equation may be used for the reduction of the observations; but if the instrument has been frequently reversed during the progress of the work, it will be more convenient to combine in one computation consecutive observations in its positive and negative positions. Employing the subscripts 1 and 2 to distinguish observations made in these respective positions, we obtain, by taking the mean of the resulting equations, Circle W. and Circle E., and introducing a correction for diurnal aberration,

$$A'=A+\frac{k}{2}\left\{(S_2-M_2)-(S_1-M_1)+\ \sigma\ (S_2-S_1)\right\}$$
$$-b\tan h+\text{Di. Ab.} \quad (164)$$

In this equation A represents the mean of the azimuths of the star at the several times of observation, and for this mean there may usually be substituted the azimuth corresponding to the mean of the times (see § 68). The level constant, b, represents the mean of the inclinations of the horizontal axis in the two positions of the instrument, and it should be noted that if the level is left undisturbed upon the axis during the reversal, the resulting bubble readings, Circle W. and Circle E., will give this mean inclination, free from the effect of inequality of pivots. In the case of a hanging level all necessity for lifting it from the axis or in any way disturbing its relation to the instrument is thus removed.

85. Diurnal Aberration.—In explanation of the last term of Equation 164 we note that the precision attain-

able with a transit instrument is sufficient to demand a consideration of the effect of diurnal aberration, and the student may show from the data in § 27 that for any star near the pole this effect is fully compensated by adding to the computed azimuth of the mark the correction,

$$\text{Di. Ab.} = +0''.32 \cos \phi \sec h. \qquad (165)$$

Since ϕ and h are very nearly equal for close circumpolar stars, this correction is practically constant and equal to $+0''.32$.

86. Example.—*Azimuth Determination with Transit.*—The following example represents the record and reduction of a single set of azimuth observations made with the large transit instrument of the "broken" type shown in the Frontispiece. Note that the recorded sidereal times show that the observations were made at about 9 or 10 o'clock A.M., 21^h or 22^h astronomical reckoning, i.e., in broad daylight. Values of the instrumental constants and other data required for the reduction follow immediately after the record, the value of the chronometer correction, ΔT, having been determined for this purpose from time observations immediately following the azimuth work. At the time of the azimuth observations Polaris was near upper culmination, and an inspection of the micrometer readings to the star, shows that they diminish progressively for Circle W., which is therefore the positive position of the instrument and is to receive the subscript 1 in the reductions. The azimuth of Polaris is computed from Equation 124.

POLARIS AND AZIMUTH MARK.

Wednesday, May 7, 1902.

Bamberg Transit. Chronometer, S. Observer, C.

Circle.	Azimuth Mark.		Polaris.		Remarks. Levels.
	Micrometer.	Chronometer.	Micrometer.	Chronometer.	
		h. m.		h. m. s.	
E.	15.637	1 8	9.803	1 12 47	Mark very
	.622		10.019	13 26	unsteady.
	.601		.189	13 56	
	.606				W. E.
	.610				29.4 59.2
					60.4 30.8
W.	15.083	1 20	19.799	1 16 18	
	.067		.578	16 59.5	+1.3
	.061		.394	17 35	
	.064				$h=44°\ 18'$
	.087				

Instrumental Constants.

$\Delta T=+31^s.0$ $k=57''.570$ $d=0''.50$

REDUCTION.

M_2	15.615	ϕ	43° 4′ 37″.0
S_2	10.004	δ	88 47 1.7
S_1	19.590		h. m. s.
M_1	15.07	a	1 23 5.4
		$T+\Delta T$	1 15 41.2
$\log(S_2-S_1)$	0.9816n	t	23 52 35.8
$\log\sigma$	9.5990	t	358 8 57.0
$+(S_2-M_2)$	−5.611	$\cos t$	9.999774
$-(S_1-M_1)$	−4.518	$\tan\phi$	9.970825
$+\sigma(S_2-S_1)$	−3.807	$\cot\delta$	8.326946
Sum	−13.936	$\sec\phi$	0.136417
		$\sin t$	8.509169n
log Sum	1.14414n	$F\begin{cases}B_1\\B_2\end{cases}$	0.008532
$\log\frac{1}{2}k$	1.45917		171
		$-\tan A$	6.981235n
$\cos\phi$	9.863		
$\sec h$	0.147	$\log x$	8.297545
0.32	9.505	$\log x^2$	6.5951
Di. Ab.	9.515		

A	180° 3′ 17″.54
Micrometer	−6 41 .15
Level	−0 0 .65
Di. Aberration	+0 0 .33
Azimuth	179 56 36 .07

To eliminate any error that might exist in the assumed value of a revolution of the micrometer screw, a second set of comparisons of the star and mark was made a half-hour later than those reduced above, when the star was on the opposite side of the mark and at an approximately equal distance from it. The resulting value of the azimuth of the mark was $A' = 179° 56' 36''.20$.

When the highest accuracy is required a considerable number of such sets of observations should be made, extending over at least three or four days and, when possible, so timed that the star will be observed at opposite points of its diurnal path, i.e., near its upper and lower culmination, in order to eliminate errors in its assumed right ascension and declination. A study of the errors of the micrometer screw should also be made (see § 72), and the resulting corrections for periodic and progressive error applied to the several readings. The azimuth of the star should, in general, be computed with six-place logarithmic tables, but when, as in this case, the star is very near the meridian five places of decimals are quite sufficient.

Query.—Is it legitimate in this case to neglect the corrections, ΔA_0, represented by Equation 127?

For an extended treatise showing the methods used in the U. S. Coast and Geodetic Survey for the determination of time and azimuth with a transit instrument, reference may be made to Appendix 7, Annual Report of the Survey for 1897–98.

REFERENCE WORKS.

For a more detailed treatment of the problems of spherical and practical astronomy than is contained in the preceding pages, the advanced student may consult with profit the following works:

1. Chauvenet. A Manual of Spherical and Practical Astronomy. 2 vols. Philadelphia. Various editions.

2. Hayford. Determination of Time, Longitude, Latitude, and Azimuth. Appendix No. 7, U. S. Coast and Geodetic Survey. Sixty-seventh Annual Report. Washington. 1899.

3. Albrecht. Formeln und Hülfstafeln für Geographische Ortsbestimmungen. Leipzig. Third Edition. 1894.

4. Albrecht. Anleitung zum Gebrauche des Zenitteleskops auf den Internationalen Breitenstationen. Berlin. 1902.

5. Bamberg. Anweisung zur Behandlung der Universal Instrumente und Theodoliten mit mikroskopischer Ablesung, etc. Berlin. 1883.

Of the above works No. 1 is the standard treatise upon the subject; an elaborate manual known and used among astronomers of every land. No. 2 is much more limited in its scope, but presents well the methods in use in the U. S. Coast and Geodetic Survey. No. 3 presents similarly the current German practice and is accompanied by a valuable series of numerical tables. No. 4 is a special monograph, and No. 5 a trade pamphlet presenting details of the use and care of geodetic instruments not readily accessible elsewhere.

TABLES

INTRODUCTION TO THE TABLES.

§ 87. The following tables are intended for use in connection with rough and approximate determinations of azimuth, latitude and time. They are sufficiently accurate for all such purposes, the quantities interpolated from each table being in general reliable to within one unit of the last decimal place there given, as is shown in the several illustrative examples. Even this degree of precision may be considerably enhanced by heeding the sign +, in many places printed after a tabular number and implying that the number thus marked is to be increased half a unit, e.g., for 9+ read 9.5. To the limit of precision thus indicated they render the observer practically independent of an almanac for the reduction of observations of stars.

For an explanation of the tabular numbers reference may be made to the sections of the text indicated above the respective tables and to the following explanation of quantities not adequately treated in the text.

Table 2, *n*. See below under Table 5.

Table 2, *V*. In addition to the use of *V* set forth in § 19 the following application may occasionally prove convenient: The solar ephemeris, i.e., the right ascension and declination of the sun, the equation of time, etc., nearly repeats itself on corresponding dates in

successive years, although, on account of the varying relation of the calendar year to the true solar year, this correspondence is not sufficiently exact to permit one almanac to be used directly as a substitute for another. By taking account of this varying relation of the calendar to the true solar year we may, however, make such use of a substitute almanac, and for that purpose we add to the calendar date for which any given quantity is to be interpolated, the longitude correction, λ, of § 17, and a further time correction entirely similar to λ, but obtained from the formula,

$$\text{Time Correction} = V_A - V_Y,$$

where V_Y denotes the tabular value of V for the given year and V_A is the value of V for the year of the almanac that is to be used. Interpolate the required quantities with the corrected time thus obtained. For example, let it be required to obtain the sun's declination, the equation of time and the right ascension of the mean sun on May 1, 1910, at 1:30 P.M. Central Standard Time, using for this purpose an almanac for the year 1905. We proceed as follows:

1905, V_A	82.691	Time corr.$+\lambda$	$+\ 0^h\ 53^m.3$
1910, V_Y	82.904	Stand. time	$1\ \ 30\ .0$
		Greenwich t.	$2\ \ 23\ .3$
$V_A - V_Y$	$-0.213d, =$	δ	$14°\ 59'\ 5''$
Time corr.	$-5^h\ 6^m.7$	E	$2\ \ 57\ .32$
λ	$+6\ \ 0\ .0$	Q	$2\ \ 35\ \ 22\ .16$

From the almanac for 1910 we find for the given time

$$\delta = 14°\ 59'\ 12''.8, \quad E = 2^m\ 57^s.16, \quad Q = 2^h\ 35^m\ 22^s.17,$$

and the discrepancies between these values and those found above fairly illustrate what may be expected in other cases.

Note that for any given meridian and almanac the Time Correction $+\lambda$, $+0^h\ 53^m.3$ in the example, is constant for a year, and that when its value has once been computed and written in the margin of the page, in all that relates to the solar ephemeris an old almanac is as convenient for use as the current one. But only quantities pertaining to the sun, e.g., δ, α, E, Q, etc., can be correctly found by the method given above.

Table 5. Right Ascensions and Declinations of the fixed stars are best obtained from the almanac of the year in which they are observed, but for the convenience of the observer not provided with such an almanac, there are here printed tables from which may be derived the coordinates of Polaris and fifty southern stars, all of which are bright enough to be readily observed in the telescope of an engineer's transit.

In order that such tables shall correctly represent the positions of the stars they must take account of three classes of variation to which the coordinates are subject, viz:

A. The Annual Variation, a nearly uniform progressive change whose amount for each right ascension and declination is shown under the heading, Ann. Var., in Table 5_A.

B. An oscillating change, aberration, solar nutation, that runs through its entire series of values each year.

C. A slower oscillating change, lunar nutation, whose period is approximately 18.7 years.

Table 5_A shows for each star, in addition to its name and stellar magnitude, § 21, its right ascension and declination on that date of the year 1900 which is printed in the next to the last column of the table. The approximate coordinates on the same date in any subsequent year, $1900+T$, may be found by adding to the tabular α_0, δ_0, T times the corresponding Ann. Var. The printed α_0, δ_0, include the effects above, designated A and B, but take no account of C, and in order to include its influence we must have recourse to Table 2 where the column n gives a correction to the time interval, T, sufficient to take into account the major part of the nutation effect, e.g., the actual α and δ are given by the equations,

$$\alpha=\alpha_0+(T+n) \text{ Ann. Var.} \qquad \delta=\delta_0+(T+n) \text{ Ann. Var.}$$

where T is always an integral number of years.

The declination thus computed may be assumed to remain constant throughout the year, and the right ascension may similarly be assumed constant for a month preceding and following the date named in the table. For remoter dates apply to the computed α a correction interpolated from Table 5_B. For an explanation of Table 5_C and the column M_0 to which it relates, see § 32.

In illustration of the foregoing we shall find the coordinates of Antares, No. 34, for May 1, 1911, as follows:

$$T+n=11-0.3=10.7 \text{ years.}$$

Epoch — May 1 − 87 days.

α_0	$16^h\ 23^m\ 19^s.5$	δ_0	$-26°\ 12'.5$
10.7 Ann. Var.	$+39\ .3$	10.7 Ann. Var.	-1.5
$\Delta\alpha$	$-\ 0\ .3$		
α	$16\ \ 23\ \ 58\ .5$	δ	$-26\ \ 14.$

The almanac furnishes as the coordinates of Antares on this date

$$\alpha = 16^h\ 23^m\ 58^s.45 \qquad \delta = -26°\ 14'\ 16''.$$

Table 6. The right ascension of Polaris is subject to such large and rapid variations that the coordinates of this star, are given under a form very different from that of Table 5. In Table 6_A there are given for the beginning of each year from 1905 to 1930 the quantities α_1, δ_1, which include the effect of the variations above designated A and C, and in Table 6_B are given at uniform intervals of 20 days throughout the years 1910 and 1930, quantities, α_2, δ_2, which represent the effect of B. Interpolate for the given year and day the several quantities α_1, δ_1, α_2, δ_2, (double interpolation in 6_B) and find the coordinates of Polaris through the relations:

$$\alpha = \alpha_1 + \alpha_2 \qquad \delta = \delta_1 + \delta_2.$$

To facilitate interpolation in 6_A there is added to 6_B a column, τ, showing the fraction of a year that has elapsed at each date.

For example, we find for Greenwich mean noon, October 15, 1905, $\tau = 0.79$,

α_1	$1^h\ 24^m\ 0^s$	δ_1	$88°\ 47'\ 48''.4$
α_2	$2\ \ 19$	δ_2	$20\ .6$
α	$1\ \ 26\ \ 19$	δ	$88\ \ 48\ \ 9\ .$

The almanac gives for Polaris on this date,

$$\alpha = 1^h\ 26^m\ 19^s.5 \qquad \delta = 88°\ 48'\ 9''.1.$$

§ 88. **Differential Coefficients.** — Some of the ends served by the formulæ printed on p. 207 may be exemplified as follows: Referring to the method of determining latitude represented by Eq. 35, it is apparent that if the altitude, h, be measured too great by 1′ the computed latitude, ϕ, will be found 1′ too small. In the notation of the differential calculus this relation is represented by the equation

$$d\phi/dh = -1,$$

a relation which in this simple form holds only when the body is in the meridian. In Eqs. 36, 37, 38, if h be measured 1′ too great the computed azimuth, A, and time, T, will be to some extent vitiated, but the amount of error thus produced is not immediately evident. The ratio of the resulting errors, dT, dA, to the dh that produced them is, however, given by the differential coefficients $\frac{dT}{dh}$, $\frac{dA}{dh}$, and the analytical equivalents of these expressions as well as others of frequent use, are given on p. 207. By a proper use of these expressions a correction for small errors, e.g., 0′ to 5′, whose presence in the data is detected only after a long reduction has been made, may be readily computed without the necessity for a second reduction of the observations; thus, in the example solved in § 30, if it be desired to substitute in place of the latitude there employed an improved value 0′.4 greater, we may find the resulting change in A_M to be

$$\Delta A_M = \frac{dA}{d\phi}(+0'.4) = -0'.4 \sec\phi \cot t = -0'.3,$$

using for sec ϕ and cot t the numerical values furnished by the original reduction.

An equally important use of these differential coefficients is that they furnish criteria by which to distinguish between favorable and unfavorable conditions for observations of a given type. Thus if it be desired to determine time from an observed altitude of a star, the conditions should be so chosen that any small error in the observed altitude shall have a minimum effect upon the concluded T, or the hour angle, t, from which T is determined. We find from the first of the differential coefficients on p. 207.

$$\frac{dt}{dh} = -\frac{1}{\cos\phi \sin A},$$

which for any given value of the latitude, ϕ, has its minimum value when $\sin A = 1$, i.e., the observation should be made upon a star near the prime vertical. Let the student show from the fourth of the differential coefficients that in the determination last considered a small error in the adopted latitude will also have its minimum effect upon the resulting chronometer correction if the observation is made upon a body due east or due west.

$\frac{dh}{dt} = -\cos\phi \sin A$ .... ϕ and δ **constant**

$\frac{dh}{d\phi} = -\cos A$ .... δ and t "

$\frac{dh}{dA} = -\cos h \tan q$ .... ϕ and δ "

$\frac{d\phi}{dt} = -\cos\phi \tan A$ .... δ and h "

$\frac{d\delta}{dt} = +\cos\delta \tan q$ .... ϕ and h "

$\frac{d\delta}{dA} = +\cos\phi \sin t$ .... ϕ and h "

$\frac{dA}{d\phi} = -\sec\phi \cot t$ .... δ and h "

$\frac{dA}{dt} = +\cos\delta \sec h \cos q$ .... ϕ and δ "

$= \sin\phi + \cos\phi \tan h \cos A$

$\frac{dq}{dt} = +\cos\phi \sec h \cos A$ .... ϕ and δ "

$\frac{d^2h}{dt^2} = -\cos\phi \cos\delta \sec h \cos A \cos q$ .... ϕ and δ "

$= -\cos\phi \cos A \cdot \frac{dA}{dt}$

$\frac{d^2A}{dt^2} = -\frac{\cos\phi}{\cos h}\sin\delta \sin A - \left(\frac{\cos\phi}{\cos h}\right)^2 \sin 2A \cdot$ ϕ and δ "

$= -\cos\delta \cos\phi \sec^2 h \{\sin h \sin A \cos q + \cos A \sin q\}$

ALTITUDE NUMBERS.

TABLE 1$_A$.—VALUES OF f. § 32.

h	0°	10°	20°	30°	40°	50°	60°		h
0°	0.766	0.778	0.815	0.885	1.000	1.192	1.532	100	0°
2	0.767	0.783	0.826	0.903	1.031	1.245	1.632	116	2
4	0.768	0.790	0.839	0.924	1.065	1.303	1.748	136	4
6	0.770	0.797	0.852	0.947	1.103	1.370	1.884	161	6
8	0.773	0.806	0.868	0.972	1.145	1.445	2.045	195	8
10	0.778	0.815	0.885	1.000	1.192	1.532	2.240		10

TABLE 1$_B$.—REFRACTION, ETC. §§ 29, 32

h'	R	R'
10°	5'.1	5'.0
20	2 .5	2 .4
30	1 .6	1 .5
40	1 .1	1 .0
50	0 .8	0 .7
60	0 .5	0 .5
70	0 .3	0 .3
80	0 .2	0 .2
90	0 .0	0 .0

YEAR NUMBERS.

TABLE 2. §§ 19, 32, 87.

Year.	V	n	Y	F
	d	y	m	
1905	82.691	−0.1	+14	1.020
1906	82.935	0.2	12	1.016
1907	83.178	0.3	11	1.012
1908	82.420	0.3	13	1.007
1909	82.663	0.3	12	1.002
1910	82.904	0.3	+10	0.997
1911	83.146	0.3	9	0.992
1912	82.386	0.2	12	0.987
1913	82.627	−0.1	10	0.982
1914	82.868	+0.1	9	0.978
1915	83.110	0.2	+ 7	0.973
1916	82.350	0.3	9	0.968
1917	82.592	0.3	8	0.964
1918	82.834	0.3	6	0.960
1919	83.076	0.3	5	0.956
1920	82.319	0.3	+ 7	0.952
1921	82.562	0.2	6	0.948
1922	82.806	+0.1	4	0.944
1923	83.050	−0.0	3	0.940
1924	82.293	0.2	5	0.936
1925	82.537	0.3	+ 4	0.932
1926	82.779	0.3	3	0.927
1927	83.022	0.3	1	0.923
1928	82.264	0.3	4	0.919
1929	82.596	0.3	3	0.914
1930	82.747	−0.2	+ 1	0.909

DAY NUMBERS.

TABLE 3.

§ 32.

For Greenwich Mean Noon.

Day of the Month.	Day of the Year.	D		ΔF
Jan. 6*	7	17^h	23^m	−0.002
21*	22	18	22	0.002
Feb. 5*	37	19	22	0.002
20*	52	20	21	0.001
Mar. 7	67	21	21	−0.001
22	82	22	20	0.000
Apr. 6	97	23	20	+0.001
21	112	0	19	0.001
May 6	127	1	18	0.002
21	142	2	17	0.003
June 5	157	3	16	0.004
20	172	4	15	0.005
July 5	187	5	14	0.005
20	202	6	13	0.005
Aug. 4	217	7	11	0.004
19	232	8	10	0.004
Sept. 3	247	9	9	0.003
18	262	10	8	0.002
Oct. 3	277	11	7	+0.000
18	292	12	6	−0.001
Nov. 2	307	13	5	0.002
17	322	14	4	0.003
Dec. 2	337	15	4	0.004
17	352	16	3	0.005
32	367	17	2	−0.006

PP for *D*.

1	$3^m.9$
2	7 .9
3	11 .8
4	15 .8
5	19 .7
6	23 .7
7	27 .6
8	31 .5
9	35 .5
10	39 .4

* In leap year subtract one day from the given date in January and February before interpolating from this table.

Year.	19
Y	
F	
R'	

h'	*Ff*

POLARIS ORIENTATION.

TABLE 4. §§ 32, 33.

t	a_0	Var. per 1^m	b_0	Var. per 1^m	t
h. m.	′	′	′	′	h. m.
0 0	− 0 +	0.40	+70+	0.00	24 0
0 30	12	0.39	70	0.04	23 30
1 0	24	0.38	68	0.08	23 0
1 30	35	0.37	65	0.12	22 30
2 0	−46 +	0.35	+61 +	0.15	22 0
2 30	56	0.32	56	0.18	21 30
3 0	65	0.29	50	0.21	21 0
3 30	73	0.25	43	0.24	20 30
4 0	−80 +	0.20	+35+	0.27	20 0
4 30	85	0.15	27	0.29	19 30
5 0	89	0.10	18	0.30	19 0
5 30	91	0.05	9	0.30	18 30
6 0	−92 +	0.00	± 0±	0.30	18 0
6 30	91	0.05	9	0.30	17 30
7 0	89	0.10	18	0.30	17 0
7 30	85	0.15	27	0.29	16 30
8 0	−80 +	0.20	−35−	0.27	16 0
8 30	73	0.25	43	0.24	15 30
9 0	65	0.29	50	0.21	15 0
9 30	56	0.32	56	0.18	14 30
10 0	−46 +	0.35	−61−	0.15	14 0
10 30	35	0.37	65	0.12	13 30
11 0	24	0.38	68	0.08	13 0
11 30	12	0.39	70	0.04	12 30
12 0	− 0 +	0.40	−70−	0.00	12 0

$t = M + Y + D.$ $A = 180° + Ffa_0.$

$h = \phi + Fb_0.$ $\phi = h' - R' - Fb_0.$

TIME STARS.

TABLE 5_A. §§ 32, 87.

No.	Star.	Mag.	α_1 1900.	Ann. Var.	δ_1 1900.	Ann. Var.	Date.	M_0
			h. m. s.	s.	° ′	′		h.m.
1	ι Ceti	3	0 14 23	+3.06	− 9 22+	+0.33	Nov. 24	8 0
2	β Ceti	2	0 38 37+	3.01	−18 32	0.33	30	0
3	θ Ceti	4	1 19 5	3.00	− 8 41+	0.31	Dec. 10	1
4	α Piscium	4	1 56 56	3.10	+ 2 17	0.29	20	0
5	δ Ceti	4	2 34 25	3.07	− 0 6	0.26	29	1
6	α Ceti	3	2 57 4	+3.13	+ 3 42	+0.24	Jan. 4	8 0
7	ε Eridani	4	3 28 14	2.82	− 9 48	0.21	11	3
8	γ Eridani	3	3 53 22+	2.80	−13 48	0.17	18	1
9	ν Eridiani	4	4 31 20	3.00	− 3 33+	0.13	27	3
10	β Orionis	0	5 9 45	2.88	− 8 19	0.07	Feb. 6	2
11	κ Orionis	2	5 43 2	+2.84	− 9 42+	+0.02	15	8 0
12	β Can. Maj.	2	6 18 19	2.64	−17 54+	−0.02	24	0
13	Sirius	−1	6 40 45+	2.64	−16 35	0.08	Mar. 1	3
14	γ Can. Maj.	4	6 59 15	2.71	−15 29+	0.08	6	1
15	η Can. Maj.	2	7 20 9+	2.37	−29 7	0.11	11	2
16	ρ Argus	3	8 3 18	+2.55	−24 1	−0.17	22	8 2
17	12 Hydræ	4	8 41 40+	2.84	−13 11	0.22	Apr. 1	1
18	α Hydræ	2	9 22 42	2.95	− 8 14	0.26	11	3
19	39 Hydræ	4	9 46 41	2.88	−14 23	0.28	17	3
20	λ Hydræ	4	10 5 44	2.92	−11 52	0.30	22	2
21	ν Hydræ	3	10 44 43	+2.96	−15 40+	−0.31	May 2	8 2
22	δ Crateris	4	11 14 22	3.00	−14 14+	0.32	10	0
23	β Virginis	4	11 45 31	3.13	+ 2 20	0.34	18	0
24	γ Corvi	3	12 10 41+	3.08	−16 59+	0.33	24	1
25	γ Virginis	3	12 36 37	3.04	− 0 54	0.33	31	0
26	θ Virginis	4	13 4 48	+3.10	− 5 0+	−0.32	June 7	8 0
27	Spica	1	13 19 57+	3.16	−10 38+	0.31	11	0
28	ζ Virginis	3	13 29 38	3.05	− 0 5+	0.31	13	1
29	π Hydræ	4	14 0 42+	3.41	−26 12	0.29	21	1
30	μ Virginis	4	14 37 49+	3.16	− 5 13+	0.26	30	2
31	β Libræ	3	15 11 40	+3.22	− 9 1	−0.22	July 9	8 1
32	μ Serpentis	4	15 44 26+	3.13	− 3 7+	0.18	17	2
33	β Scorpii	3	15 59 40	3.48	−19 32	0.17	21	2
34	Antares	1	16 23 19+	3.67	−26 12+	0.14	27	2
35	η Ophiuchi	3	17 4 41	3.44	−15 36	0.08	Aug. 7	0

TIME STARS.

TABLE 5A. §§ 32, 87.

No.	Star.	Mag.	α_1 1900.	Ann. Var.	δ_1 1900.	Ann. Var.	Date.	M_0
			h. m. s.	s	° ′	′		h.m.
36	ξ Serpentis..	4	17 31 54+	+3.43	−15 20	−0.04	Aug. 13	8 3
37	ν Ophiuchi .	4	17 53 34	3.30	− 9 45+	−0.01	19	1
38	η Serpentis..	3	18 16 10+	3.10	− 2 55+	0.00	25	0
39	σ Sagittarii..	2	18 49 7	3.72	−26 25	+0.07	Sept. 2	2
40	ρ Sagittarii..	4	19 15 55+	3.48	−18 2	+0.11	9	1
41	η Aquilæ . . .	4	19 47 25+	+3.06	+ 0 45	+0.15	17	8 1
42	β Capricorni	3	20 15 26+	3.37	−15 5+	0.18	24	1
43	ε Aquarii . . .	4	20 42 19	3.25	− 9 51+	0.22	Oct. 1	0
44	ν Aquarii . . .	4	21 4 12	3.27	−11 46+	0.24	6	2
45	β Aquarii . . .	3	21 26 20+	3.16	− 6 0	0.26	12	1
46	α Aquarii . . .	3	22 0 42	+3.08	− 0 48	+0.29	21	8 0
47	ζ Aquarii . . .	4	22 23 44	3.09	− 0 31+	0.30	26	3
48	δ Aquarii . . .	3	22 49 23+	3.19	−16 21	0.32	Nov. 2	1
49	γ Piscium. . .	4	23 12 2	3.11	+ 2 44+	0.33	8	0
50	ω Aquarii . . .	4	23 37 35+	3.11	−15 5+	0.33	14	2

TABLE 5B. § 87.

Correction to R.A.

Date.	$\Delta\alpha$
	s.
150 days before	−1.9
100 " "	−0.5
50 " "	+0.3
Epoch	0.0
50 days after. .	−0.7

TABLE 5C. § 32.

Correction to M_0.

Longitude.	Leap Year. Jan. Feb.	Leap Year. Mch. Dec.	I.	II.	III.
	m.	m.	m.	m.	m.
12^h E.	8	4	5	6	7
6 E.	7	3	4	5	6
Greenwich	6	2	3	4	5
6^h W.	5	1	2	3	4
12 W.	4	0	1	2	3

POLARIS.

TABLE 6_A. § 87.

Year.	α_1 h	m	s		δ_1 °	′	″	
1905	1	23	45	19	88	47	35	17
1906		24	4	18		47	52	18
1907		24	22	17		48	10	19
1908		24	39	17		48	29	19
1909		24	56	19		48	48+	20
1910	1	25	15	22	88	49	9	20
1911		25	37	25		49	29+	21
1912		26	2	28		49	50+	21
1913		26	30	31		50	11+	21
1914		27	1	35		50	32	20
1915	1	27	36	37	88	50	52+	20
1916		28	13	39		51	12	18
1917		28	52	39		51	30+	18
1918		29	31	39		51	48	17
1919		30	10	37		52	5	16
1920	1	30	47	34	88	52	21+	16
1921		31	21	31		52	37+	16
1922		31	52	28		52	53+	16
1923		32	20	24		53	9+	17
1924		32	44	23		53	26+	17
1925	1	33	7	21	88	53	43+	19
1926		33	28	20		54	2	19
1927		33	48	22		54	21	20
1928		34	10	25		54	40+	20
1929		34	35	27		55	1	20
1930	1	35	2	31	88	55	21+	21
1931		35	33	35		55	42	20
1932		36	8			56	2+	

POLARIS.

TABLE 6_B. § 87.

For Greenwich Mean Noon.

Day.	τ	α_2 1910	α_2 1930	δ_2 1910	δ_2 1930
Jan. 1	0.00	$+96^s$	$+101^s$	$+40''$	$+40''$
21	0.06	74	77	40	40
Feb. 10	0.11	52	53	38	38
Mar. 2	0.16	33	32	33	34
22	0.22	21	17	27	28
Apr. 11	0.27	17	11	20	21
May 1	0.33	20	14	13	14
21	0.38	31	24	7	8
June 10	0.44	46	41	2	3
30	0.49	65	62	0	1
July 20	0.55	86	84	0	0
Aug. 9	0.60	105	105	2	2
29	0.66	122	123	6	6
Sept. 18	0.71	134	137	12	11
Oct. 8	0.77	140	145	18	17
28	0.82	140	147	25	24
Nov. 17	0.88	134	140	31	30
Dec. 7	0.93	121	127	36	36
27	0.98	102	107	40	39

$$\text{R. A.} = \alpha_1 + \alpha_2.$$
$$\text{Dec.} = \delta_1 + \delta_2.$$

INDEX.

SHORT-TITLE CATALOGUE

OF THE

PUBLICATIONS

OF

JOHN WILEY & SONS,

NEW YORK.

LONDON: CHAPMAN & HALL, LIMITED.

ARRANGED UNDER SUBJECTS.

Descriptive circulars sent on application. Books marked with an asterisk (*) are sold at *net* prices only. All books are bound in cloth unless otherwise stated.

AGRICULTURE—HORTICULTURE—FORESTRY.

Armsby's Principles of Animal Nutrition 8vo, $4 00
Budd and Hansen's American Horticultural Manual:
    Part I. Propagation, Culture, and Improvement..................12mo, 1 50
    Part II. Systematic Pomology.......................................12mo, 1 50
Elliott's Engineering for Land Drainage.......................................12mo, 1 50
    Practical Farm Drainage 2d Edition, Rewritten..................12mo, 1 50
Graves's Forest Mensuration..8vo, 4 00
Green's Principles of American Forestry.....................................12mo, 1 50
Grotenfelt's Principles of Modern Dairy Practice. (Woll)..............12mo, 2 00
* Herrick's Denatured or Industrial Alcohol...................................8vo, 4 00
Kemp and Waugh's Landscape Gardening. New Edition, Rewritten. (In Preparation.)
* McKay and Larsen's Principles and Practice of Butter-making 8vo, 1 50
Maynard's Landscape Gardening as Applied to Home Decoration........12mo, 1 50
Quaintance and Scott's Insects and Diseases of Fruits. (In Preparation).
Sanderson's Insects Injurious to Staple Crops..............................12mo, 1 50
* Schwarz's Longleaf Pine in Virgin Forests..................................12mo, 1 25
Stockbridge's Rocks and Soils...8vo, 2 50
Winton's Microscopy of Vegetable Foods.......................................8vo, 7 50
Woll's Handbook for Farmers and Dairymen..................................16mo, 1 50

ARCHITECTURE.

Baldwin's Steam Heating for Buildings..12mo, 2 50
Berg's Buildings and Structures of American Railroads.....................4to, 5 00
Birkmire's Architectural Iron and Steel..8vo, 3 50
    Compound Riveted Girders as Applied in Buildings.................8vo, 2 00
    Planning and Construction of American Theatres...................8vo, 3 00
    Planning and Construction of High Office Buildings...............8vo, 3 50
    Skeleton Construction in Buildings......................................8vo, 3 00
Briggs's Modern American School Buildings...................................8vo, 4 00
Byrne's Inspection of Material and Wormanship Employed in Construction. 16mo, 3 00
Carpenter's Heating and Ventilating of Buildings...........................8vo, 4 00
* Corthell's Allowable Pressure on Deep Foundations......................12mo, 1 25

Freitag's Architectural Engineering. 8vo 3 50
Fireproofing of Steel Buildings. 8vo, 2 50
French and Ives's Stereotomy. 8vo, 2 50
Gerhard's Guide to Sanitary House-Inspection. 16mo, 1 00
* Modern Baths and Bath Houses. 8vo, 3 00
Sanitation of Public Buildings 12mo, 1 50
Theatre Fires and Panics. 12mo, 1 50
Holley and Ladd's Analysis of Mixed Paints, Color Pigments, and Varnishes
Large 12mo, 2 50
Johnson's Statics by Algebraic and Graphic Methods 8vo, 2 00
Kellaway's How to Lay Out Suburban Home Grounds 8vo, 2 00
Kidder's Architects' and Builders' Pocket-book. . . . . .16mo, mor. 5 00
Maire's Modern Pigments and their Vehicles 12mo, 2 00
Merrill's Non-metallic Minerals: Their Occurrence and Uses. 8vo, 4 00
Stones for Building and Decoration. 8vo, 5 00
Monckton's Stair-building. 4to, 4 00
Patton's Practical Treatise on Foundations. 8vo, 5 00
Peabody's Naval Architecture 8vo, 7 50
Rice's Concrete-block Manufacture 8vo, 2 00
Richey's Handbook for Superintendents of Construction. . . . . .16mo, mor. 4 00
* Building Mechanics' Ready Reference Book:
* Building Foreman's Pocket Book and Ready Reference. (In Preparation.)
* Carpenters' and Woodworkers' Edition. . . . . .16mo, mor. 1 50
* Cement Workers and Plasterer's Edition. . . . . .16mo, mor. 1 50
* Plumbers', Steam-Filters', and Tinners' Edition. . . .16mo, mor. 1 50
* Stone- and Brick-masons' Edition. . . . . .16mo, mor. 1 50
Sabin's House Painting 12mo, 1 00
Industrial and Artistic Technology of Paints and Varnish. 8vo, 3 00
Siebert and Biggin's Modern Stone-cutting and Masonry. 8vo, 1 50
Snow's Principal Species of Wood. 8vo, 3 50
Towne's Locks and Builders' Hardware. . . . . .18mo, mor. 3 00
Wait's Engineering and Architectural Jurisprudence 8vo, 6 00
Sheep, 6 50
Law of Contracts. 8vo, 3 00
Law of Operations Preliminary to Construction in Engineering and Architecture. 8vo, 5 00
Sheep, 5 50
Wilson's Air Conditioning. . . . . .12mo, 1 5'
Worcester and Atkinson's Small Hospitals, Establishment and Maintenance, Suggestions for Hospital Architecture, with Plans for a Small Hospital.
12mo, 1 25

ARMY AND NAVY.

Bernadou's Smokeless Powder, Nitro-cellulose, and the Theory of the Cellulose Molecule. 12mo, 2 50
Chase's Art of Pattern Making. 12mo, 2 50
Screw Propellers and Marine Propulsion. 8vo, 3 00
Cloke's Enlisted Specialist's Examiner. (In Press.)
Gunner's Examiner 8vo, 1 50
Craig's Azimuth. 4to, 3 50
Crehore and Squier's Polarizing Photo-chronograph. 8vo, 3 00
* Davis's Elements of Law. 8vo, 2 50
* Treatise on the Military Law of United States. 8vo, 7 00
Sheep, 7 50
De Brack's Cavalry Outpost Duties. (Carr). . . . . .24mo, mor. 2 00
* Dudley's Military Law and the Procedure of Courts-martial. . Large 12mo, 2 50
Durand's Resistance and Propulsion of Ships. 8vo, 5 00

* Dyer's Handbook of Light Artillery	12mo,	
Eissler's Modern High Explosives	8vo,	4 00
* Fiebeger's Text-book on Field Fortification	Large 12mo,	2 00
Hamilton and Bond's The Gunner's Catechism	18mo,	1 00
* Hoff's Elementary Naval Tactics	8vo,	1 50
Ingalls's Handbook of Problems in Direct Fire	8vo,	4 00
* Lissak's Ordnance and Gunnery	8vo,	6 00
* Ludlow's Logarithmic and Trigonometric Tables	8vo,	1 00
* Lyons's Treatise on Electromagnetic Phenomena. Vols. I. and II.	8vo, each,	6 00
* Mahan's Permanent Fortifications. (Mercur)	8vo, half mor.	7 50
Manual for Courts-martial	16mo, mor.	1 50
* Mercur's Attack of Fortified Places	12mo,	2 00
* Elements of the Art of War	8vo,	4 00
Metcalf's Cost of Manufactures—And the Administration of Workshops	8vo,	5 00
Nixon's Adjutants' Manual	24mo,	1 00
Peabody's Naval Architecture	8vo,	7 50
* Phelps's Practical Marine Surveying	8vo,	2 50
Putnam's Nautical Charts. (In Press.)		
Sharpe's Art of Subsisting Armies in War	18mo, mor.	1 50
* Tupes and Poole's Manual of Bayonet Exercises and Musketry Fencing.	24mo, leather,	50
* Weaver's Military Explosives	8vo,	3 00
Woodhull's Notes on Military Hygiene	16mo,	1 50

ASSAYING.

Betts's Lead Refining by Electrolysis	8vo,	4 00
Fletcher's Practical Instructions in Quantitative Assaying with the Blowpipe.	16mo, mor.	1 50
Furman's Manual of Practical Assaying	8vo,	3 00
Lodge's Notes on Assaying and Metallurgical Laboratory Experiments	8vo,	3 00
Low's Technical Methods of Ore Analysis	8vo,	3 00
Miller's Cyanide Process	12mo,	1 00
Manual of Assaying	12mo,	1 00
Minet's Production of Aluminum and its Industrial Use. (Waldo)	12mo,	2 50
O'Driscoll's Notes on the Treatment of Gold Ores	8vo,	2 00
Ricketts and Miller's Notes on Assaying	8vo,	3 00
Robine and Lenglen's Cyanide Industry. (Le Clerc)	8vo,	4 00
Ulke's Modern Electrolytic Copper Refining	8vo,	3 00
Wilson's Chlorination Process	12mo,	1 50
Cyanide Processes	12mo,	1 50

ASTRONOMY.

Comstock's Field Astronomy for Engineers	8vo,	2 50
Craig's Azimuth	4to,	3 50
Crandall's Text-book on Geodesy and Least Squares	8vo,	3 00
Doolittle's Treatise on Practical Astronomy	8vo,	4 00
Gore's Elements of Geodesy	8vo,	2 50
Hayford's Text-book of Geodetic Astronomy	8vo,	3 00
Merriman's Elements of Precise Surveying and Geodesy	8vo,	2 50
* Michie and Harlow's Practical Astronomy	8vo,	3 00
Rust's Ex-meridian Altitude, Azimuth and Star-Finding Tables. (In Press.)		
* White's Elements of Theoretical and Descriptive Astronomy	12mo,	2 00

CHEMISTRY.

* Abderhalden's Physiological Chemistry in Thirty Lectures. (Hall and Defren) 8vo, 5 00
* Abegg's Theory of Electrolytic Dissociation. (von Ende).......... 12mo, 1 25
Alexeyeff's General Principles of Organic Syntheses. (Matthews)........ 8vo, 3 00
Allen's Tables for Iron Analysis.................................. 8vo, 3 00
Arnold's Compendium of Chemistry. (Mandel)............... Large 12mo, 3 50
Association of State and National Food and Dairy Departments, Hartford, Meeting, 1906.. 8vo, 3 00
Jamestown Meeting, 1907.. 8vo, 3 00
Austen's Notes for Chemical Students................................ 12mo, 1 50
Baskerville's Chemical Elements. (In Preparation.)
Bernadou's Smokeless Powder.—Nitro-cellulose, and Theory of the Cellulose Molecule.. 12mo, 2 50
* Blanchard's Synthetic Inorganic Chemistry........................ 12mo, 1 00
* Browning's Introduction to the Rarer Elements.................... 8vo, 1 50
Brush and Penfield's Manual of Determinative Mineralogy............ 8vo, 4 00
* Claassen's Beet-sugar Manufacture. (Hall and Rolfe)................ 8vo, 3 00
Classen's Quantitative Chemical Analysis by Electrolysis. (Boltwood)... 8vo, 3 00
Cohn's Indicators and Test-papers.................................. 12mo, 2 00
Tests and Reagents.. 8vo, 3 00
* Danneel's Electrochemistry. (Merriam)............................ 12mo, 1 25
Duhem's Thermodynamics and Chemistry. (Burgess).................. 8vo, 4 00
Eakle's Mineral Tables for the Determination of Minerals by their Physical Properties.. 8vo, 1 25
Eissler's Modern High Explosives.................................. 8vo, 4 00
Effront's Enzymes and their Applications. (Prescott)................ 8vo, 3 00
Erdmann's Introduction to Chemical Preparations. (Dunlap).......... 12mo, 1 25
* Fischer's Physiology of Alimentation..................... Large 12mo, 2 00
Fletcher's Practical Instructions in Quantitative Assaying with the Blowpipe. 12mo, mor. 1 50
Fowler's Sewage Works Analyses.................................. 12mo, 2 00
Fresenius's Manual of Qualitative Chemical Analysis. (Wells).......... 8vo, 5 00
Manual of Qualitative Chemical Analysis. Part I. Descriptive. (Wells) 8vo, 3 00
Quantitative Chemical Analysis. (Cohn) 2 vols................. 8vo, 12 50
When Sold Separately, Vol. I, $6. Vol. II, $8.
Fuertes's Water and Public Health.................................. 12mo, 1 50
Furman's Manual of Practical Assaying.............................. 8vo, 3 00
* Getman's Exercises in Physical Chemistry.......................... 12mo, 2 00
Gill's Gas and Fuel Analysis for Engineers.......................... 12mo, 1 25
* Gooch and Browning's Outlines of Qualitative Chemical Analysis. Large 12mo, 1 25
Grotenfelt's Principles of Modern Dairy Practice. (Woll).............. 12mo, 2 00
Groth's Introduction to Chemical Crystallography (Marshall).......... 12mo, 1 25
Hammarsten's Text-book of Physiological Chemistry. (Mandel)........ 8vo, 4 00
Hanausek's Microscopy of Technical Products. Winton)................ 8vo, 5 00
* Haskins and Macleod's Organic Chemistry.......................... 12mo, 2 00
Helm's Principles of Mathematical Chemistry. (Morgan)............ 12mo, 1 50
Hering's Ready Reference Tables (Conversion Factors)........... 16mo, mor. 2 50
* Herrick's Denatured or Industrial Alcohol.......................... 8vo, 4 00
Hinds's Inorganic Chemistry.. 8vo, 3 00
* Laboratory Manual for Students................................ 12mo, 1 00
* Holleman's Laboratory Manual of Organic Chemistry for Beginners. (Walker).. 12mo, 1 00
Text-book of Inorganic Chemistry. (Cooper).................... 8vo, 2 50
Text-book of Organic Chemistry. (Walker and Mott)............ 8vo, 2 50
Holley and Ladd's Analysis of Mixed Paints, Color Pigments, and Varnishes. Large 12mo, 2 50

Hopkins's Oil-chemists' Handbook....8vo, 3 00

Iddings's Rock Minerals....8vo, 5 00

Jackson's Directions for Laboratory Work in Physiological Chemistry..8vo, 1 25

Johannsen's Determination of Rock-forming Minerals in Thin Sections...8vo, 4 00

Johnson's Chemical Analysis of Special Steels. (In Preparation.)

Keep's Cast Iron....8vo, 2 50

Ladd's Manual of Quantitative Chemical Analysis....12mo, 1 00

Landauer's Spectrum Analysis. (Tingle)....8vo, 3 00

* Langworthy and Austen's Occurrence of Aluminium in Vegetable Products, Animal Products, and Natural Waters....8vo, 2 00

Lassar-Cohn's Application of Some General Reactions to Investigations in Organic Chemistry. (Tingle)....12mo, 1 00

Leach's Inspection and Analysis of Food with Special Reference to State Control....8vo, 7 50

Löb's Electrochemistry of Organic Compounds. (Lorenz)....8vo, 3 00

Lodge's Notes on Assaying and Metallurgical Laboratory Experiments....8vo, 3 00

Low's Technical Method of Ore Analysis....8vo, 3 00

Lunge's Techno-chemical Analysis. (Cohn)....12mo, 1 00

* McKay and Larsen's Principles and Practice of Butter-making....8vo, 1 50

Maire's Modern Pigments and their Vehicles....12mo, 2 00

Mandel's Handbook for Bio-chemical Laboratory....12mo, 1 50

* Martin's Laboratory Guide to Qualitative Analysis with the Blowpipe..12mo, 60

Mason's Examination of Water. (Chemical and Bacteriological.)....12mo, 1 25

Water-supply. (Considered Principally from a Sanitary Stan dpi 8vo, 4 00

Matthews's Textile Fibres. 2d Edition, Rewritten....8vo, 4 00

* Meyer's Determination of Radicles in Carbon Compounds. (Tingle)..12mo, 1 25

Miller's Cyanide Process....12mo, 1 00

Manual of Assaying....12mo, 1 00

Minet's Production of Aluminum and its Industrial Use. (Waldo)....12mo, 2 50

Mixter's Elementary Text-book of Chemistry....12mo, 1 50

Morgan's Elements of Physical Chemistry....12mo, 3 00

Outline of the Theory of Solutions and its Results....12mo, 1 00

* Physical Chemistry for Electrical Engineers....12mo, 1 50

Morse's Calculations used in Cane-sugar Factories....16mo, mor. 1 50

* Muir's History of Chemical Theories and Laws....8vo, 4 00

Mulliken's General Method for the Identification of Pure Organic Compounds. Vol. I....Large 8vo, 5 00

O'Driscoll's Notes on the Treatment of Gold Ores....8vo, 2 00

Ostwald's Conversations on Chemistry. Part One. (Ramsey)....12mo, 1 50

" " " " Part Two. (Turnbull)....12mo, 2 00

* Palmer's Practical Test Book of Chemistry....12mo, 1 00

* Pauli's Physical Chemistry in the Service of Medicine. (Fischer)....12mo, 1 25

* Penfield's Notes on Determinative Mineralogy and Record of Mineral Tests. 8vo, paper, 50

Tables of Minerals, Including the Use of Minerals and Statistics of Domestic Production....8vo, 1 00

Pictet's Alkaloids and their Chemical Constitution. (Biddle)....8vo, 5 00

Poole's Calorific Power of Fuels....8vo, 3 00

Prescott and Winslow's Elements of Water Bacteriology, with Special Reference to Sanitary Water Analysis....12mo, 1 50

* Reisig's Guide to Piece-dyeing....8vo, 25 00

Richards and Woodman's Air, Water, and Food from a Sanitary Standpoint..8vo, 2 00

Ricketts and Miller's Notes on Assaying....8vo, 3 00

Rideal's Disinfection and the Preservation of Food....8vo, 4 00

Sewage and the Bacterial Purification of Sewage....8vo, 4 00

Riggs's Elementary Manual for the Chemical Laboratory....8vo, 1 25

Robine and Lenglen's Cyanide Industry. (Le Clerc)....8vo, 4 00

Ruddiman's Incompatibilities in Prescriptions....8vo, 2 00

Whys in Pharmacy....12mo, 1 00

Ruer's Elements of Metallography. (Mathewson) (In Preparation.)
Sabin's Industrial and Artistic Technology of Paints and Varnish........8vo, 3 00
Salkowski's Physiological and Pathological Chemistry. (Orndorff)......8vo, 2 50
Schimpf's Essentials of Volumetric Analysis........................12mo, 1 25
* Qualitative Chemical Analysis..........................8vo, 1 25
Text-book of Volumetric Analysis..........................12mo, 2 50
Smith's Lecture Notes on Chemistry for Dental Students.............8vo, 2 50
Spencer's Handbook for Cane Sugar Manufacturers.............16mo, mor. 3 00
Handbook for Chemists of Beet-sugar Houses..............16mo, mor. 3 00
Stockbridge's Rocks and Soils.....................................8vo, 2 50
* Tillman's Descriptive General Chemistry..........................8vo, 3 00
* Elementary Lessons in Heat..................................8vo, 1 50
Treadwell's Qualitative Analysis. (Hall)..........................8vo, 3 00
Quantitative Analysis. (Hall)..............................8vo, 4 00
Turneaure and Russell's Public Water-supplies.......................8vo, 5 00
Van Deventer's Physical Chemistry for Beginners. (Boltwood).......12mo, 1 50
Venable's Methods and Devices for Bacterial Treatment of Sewage......8vo, 3 00
Ward and Whipple's Freshwater Biology. (In Press.)
Ware's Beet-sugar Manufacture and Refining. Vol. I............Small 8vo, 4 00
" " " " " Vol. II..........Small 8vo, 5 00
Washington's Manual of the Chemical Analysis of Rocks..............8vo, 2 00
* Weaver's Military Explosives.....................................8vo, 3 00
Wells's Laboratory Guide in Qualitative Chemical Analysis.............8vo, 1 50
Short Course in Inorganic Qualitative Chemical Analysis for Engineering Students..12mo, 1 50
Text-book of Chemical Arithmetic.............................12mo, 1 25
Whipple's Microscopy of Drinking-water..............................8vo, 3 50
Wilson's Chlorination Process......................................12mo, 1 50
Cyanide Processes...12mo, 1 50
Winton's Microscopy of Vegetable Foods..............................8vo, 7 50

CIVIL ENGINEERING.

BRIDGES AND ROOFS. HYDRAULICS. MATERIALS OF ENGINEERING. RAILWAY ENGINEERING.

Baker's Engineers' Surveying Instruments............................12mo, 3 00
Bixby's Graphical Computing Table.................Paper 19½ × 24¼ inches. 25
Breed and Hosmer's Principles and Practice of Surveying. 2 Volumes.
Vol. I. Elementary Surveying..................................8vo, 3 00
Vol. II. Higher Surveying.....................................8vo, 2 50
* Burr's Ancient and Modern Engineering and the Isthmian Canal.....8vo, 3 50
Comstock's Field Astronomy for Engineers............................8vo, 2 50
* Corthell's Allowable Pressures on Deep Foundations................12mo, 1 25
Crandall's Text-book on Geodesy and Least Squares....................8vo, 3 00
Davis's Elevation and Stadia Tables..................................8vo, 1 00
Elliott's Engineering for Land Drainage..............................12mo, 1 50
Practical Farm Drainage....................................12mo, 1 00
*Fiebeger's Treatise on Civil Engineering............................8vo, 5 00
Flemer's Phototopographic Methods and Instruments....................8vo, 5 00
Folwell's Sewerage. (Designing and Maintenance.)....................8vo, 3 00
Freitag's Architectural Engineering..................................8vo, 3 50
French and Ives's Stereotomy...8vo, 2 50
Goodhue's Municipal Improvements.....................................12mo, 1 50
Gore's Elements of Geodesy...8vo, 2 50
* Hauch's and Rice's Tables of Quantities for Preliminary Estimates..12mo, 1 25
Hayford's Text-book of Geodetic Astronomy............................8vo, 3 00
Hering's Ready Reference Tables. (Conversion Factors).........16mo, mor. 2 50
Howe's Retaining Walls for Earth.....................................12mo, 1 25

* Ives's Adjustments of the Engineer's Transit and Level	16mo, Bds.	25
Ives and Hilts's Problems in Surveying	16mo, mor.	1 50
Johnson's (J. B.) Theory and Practice of Surveying	Small 8vo,	4 00
Johnson's (L. J.) Statics by Algebraic and Graphic Methods	8vo,	2 00
Kinnicutt, Winslow and Pratt's Purification of Sewage. (In Preparation.)		
Laplace's Philosophical Essay on Probabilities. (Truscott and Emory)	12mo,	2 00
Mahan's Descriptive Geometry	8vo,	1 50
Treatise on Civil Engineering. (1873.) (Wood)	8vo,	5 00
Merriman's Elements of Precise Surveying and Geodesy	8vo,	2 50
Merriman and Brooks's Handbook for Surveyors	16mo, mor.	2 00
Nugent's Plane Surveying	8vo,	3 50
Ogden's Sewer Construction. (In Press.)		
Sewer Design	12mo,	2 00
Parsons's Disposal of Municipal Refuse	8vo,	2 00
Patton's Treatise on Civil Engineering	8vo, half leather,	7 50
Reed's Topographical Drawing and Sketching	4to,	5 00
Rideal's Sewage and the Bacterial Purification of Sewage	8vo,	4 00
Riemer's Shaft-sinking under Difficult Conditions. (Corning and Peele)	8vo,	3 00
Siebert and Biggin's Modern Stone-cutting and Masonry	8vo,	1 50
Smith's Manual of Topographical Drawing. (McMillan)	8vo,	2 50
Soper's Air and Ventilation of Subways	Large 12mo,	2 50
Tracy's Plane Surveying	16mo, mor.	3 00
* Trautwine's Civil Engineer's Pocket-book	16mo, mor.	5 00
Venable's Garbage Crematories in America	8vo,	2 00
Methods and Devices for Bacterial Treatment of Sewage	8vo,	3 00
Wait's Engineering and Architectural Jurisprudence	8vo,	6 00
	Sheep,	6 50
Law of Contracts	8vo,	3 00
Law of Operations Preliminary to Construction in Engineering and Architecture	8vo,	5 00
	Sheep,	5 50
Warren's Stereotomy—Problems in Stone-cutting	8vo,	2 50
* Waterbury's Vest-Pocket Hand-book of Mathematics for Engineers.	$2\frac{7}{8}\times5\frac{3}{8}$ inches, mor.	1 00
Webb's Problems in the Use and Adjustment of Engineering Instruments.	16mo, mor.	1 25
Wilson's (H. N.) Topographic Surveying	8vo,	3 50
Wilson's (W. L.) Elements of Railroad Track and Construction. (In Press.)		

BRIDGES AND ROOFS.

Boller's Practical Treatise on the Construction of Iron Highway Bridges	8vo,	2 00
Burr and Falk's Design and Construction of Metallic Bridges	8vo,	5 00
Influence Lines for Bridge and Roof Computations	8vo,	3 00
Du Bois's Mechanics of Engineering. Vol. II	Small 4to,	10 00
Foster's Treatise on Wooden Trestle Bridges	4to,	5 00
Fowler's Ordinary Foundations	8vo,	3 50
French and Ives's Stereotomy	8vo,	2 50
Greene's Arches in Wood, Iron, and Stone	8vo,	2 50
Bridge Trusses	8vo,	2 50
Roof Trusses	8vo,	1 25
Grimm's Secondary Stresses in Bridge Trusses	8vo,	2 50
Heller's Stresses in Structures and the Accompanying Deformations	8vo,	3 00
Howe's Design of Simple Roof-trusses in Wood and Steel	8vo,	2 00
Symmetrical Masonry Arches	8vo,	2 50
Treatise on Arches	8vo,	4 00
Johnson, Bryan, and Turneaure's Theory and Practice in the Designing of Modern Framed Structures	Small 4to,	10 00

Merriman and Jacoby's Text-book on Roofs and Bridges:

Part I. Stresses in Simple Trusses..............................8vo, 2 50

Part II. Graphic Statics..................................8vo, 2 50

Part III. Bridge Design..................................8vo, 2 50

Part IV. Higher Structures................................8vo, 2 50

Morison's Memphis Bridge..............................Oblong 4to, 10 00

Sondericker's Graphic Statics, with Applications to Trusses, Beams, and Arches. 8vo, 2 00

Waddell's De Pontibus, Pocket-book for Bridge Engineers...... 16mo, mor, 2 00

* Specifications for Steel Bridges..........................12mo, 50

Waddell and Harrington's Bridge Engineering. (In Preparation.)

Wright's Designing of Draw-spans. Two parts in one volume.........8vo, 3 50

HYDRAULICS.

Barnes's Ice Formation..8vo, 3 00

Bazin's Experiments upon the Contraction of the Liquid Vein Issuing from an Orifice. (Trautwine)..............................8vo, 2 00

Bovey's Treatise on Hydraulics..................................8vo, 5 00

Church's Diagrams of Mean Velocity of Water in Open Channels. Oblong 4to, paper, 1 50

Hydraulic Motors..8vo, 2 00

Mechanics of Engineering..................................8vo, 6 00

Coffin's Graphical Solution of Hydraulic Problems..............16mo, mor. 2 50

Flather's Dynamometers, and the Measurement of Power............12mo, 3 00

Folwell's Water-supply Engineering..............................8vo, 4 00

Frizell's Water-power..8vo, 5 00

Fuertes's Water and Public Health..............................12mo, 1 50

Water-filtration Works......................................12mo, 2 50

Ganguillet and Kutter's General Formula for the Uniform Flow of Water in Rivers and Other Channels. (Hering and Trautwine)..........8vo, 4 00

Hazen's Clean Water and How to Get It.....................Large 12mo, 1 50

Filtration of Public Water-supplies..........................8vo, 3 00

Hazlehurst's Towers and Tanks for Water-works....................8vo, 2 50

Herschel's 115 Experiments on the Carrying Capacity of Large, Riveted, Metal Conduits..8vo, 2 00

Hoyt and Grover's River Discharge................................8vo, 2 00

Hubbard and Kiersted's Water-works Management and Maintenance.....8vo, 4 00

* Lyndon's Development and Electrical Distribution of Water Power....8vo, 3 00

Mason's Water-supply. (Considered Principally from a Sanitary Standpoint.) 8vo, 4 00

Merriman's Treatise on Hydraulics................................8vo, 5 00

* Michie's Elements of Analytical Mechanics........................8vo, 4 00

* Molitor's Hydraulics of Rivers, Weirs and Sluices..................8vo, 2 00

Schuyler's Reservoirs for Irrigation, Water-power, and Domestic Water-supply..Large 8vo, 5 00

* Thomas and Watt's Improvement of Rivers...........................4to, 6 00

Turneaure and Russell's Public Water-supplies.......................8vo, 5 00

Wegmann's Design and Construction of Dams. 5th Ed., enlarged 4to, 6 00

Water-supply of the City of New York from 1658 to 1895..........4to, 10 00

Whipple's Value of Pure Water..........................Large 12mo, 1 00

Williams and Hazen's Hydraulic Tables..............................8vo, 1 50

Wilson's Irrigation Engineering...........................Small 8vo, 4 00

Wolff's Windmill as a Prime Mover..................................8vo, 3 00

Wood's Elements of Analytical Mechanics............................8vo, 3 00

Turbines..8vo, 2 50

MATERIALS OF ENGINEERING.

Baker's Roads and Pavements.......................................8vo, 5 00
Treatise on Masonry Construction.......................................8vo, 5 00
Birkmire's Architectural Iron and Steel.......................................8vo, 3 50
Compound Riveted Girders as Applied in Buildings.......................................8vo, 2 00
Black's United States Public Works.......................................Oblong 4to, 5 00
Bleininger's Manufacture of Hydraulic Cement. (In Preparation.)
* Bovey's Strength of Materials and Theory of Structures.......................................8vo, 7 50
Burr's Elasticity and Resistance of the Materials of Engineering.......................................8vo, 7 50
Byrne's Highway Construction.......................................8vo, 5 00
Inspection of the Materials and Workmanship Employed in Construction. 16mo, 3 00
Church's Mechanics of Engineering.......................................8vo, 6 00
Du Bois's Mechanics of Engineering.
Vol. I. Kinematics, Statics, Kinetics.......................................Small 4to, 7 50
Vol. II. The Stresses in Framed Structures, Strength of Materials and Theory of Flexures.......................................Small 4to, 10 00
*Eckel's Cements, Limes, and Plasters.......................................8vo, 6 00
Stone and Clay Products used in Engineering. (In Preparation.)
Fowler's Ordinary Foundations.......................................8vo, 3 50
Graves's Forest Mensuration.......................................8vo, 4 00
Green's Principles of American Forestry.......................................12mo, 1 50
* Greene's Structural Mechanics.......................................8vo, 2 50
Holly and Ladd's Analysis of Mixed Paints, Color Pigments and Varnishes Large 12mo, 2 50
Johnson's (C. M.) Chemical Analysis of Special Steels. (In Preparation.)
Johnson's (J. B.) Materials of Construction.......................................Large 8vo, 6 00
Keep's Cast Iron.......................................8vo, 2 50
Kidder's Architects and Builders' Pocket-book.......................................16mo, 5 00
Lanza's Applied Mechanics.......................................8vo, 7 50
Maire's Modern Pigments and their Vehicles.......................................12mo, 2 00
Martens's Handbook on Testing Materials. (Henning) 2 vols.......................................8vo, 7 50
Maurer's Technical Mechanics.......................................8vo, 4 00
Merrill's Stones for Building and Decoration.......................................8vo, 5 00
Merriman's Mechanics of Materials.......................................8vo, 5 00
* Strength of Materials.......................................12mo, 1 00
Metcalf's Steel. A Manual for Steel-users.......................................12mo, 2 00
Morrison's Highway Engineering.......................................8vo, 2 50
Patton's Practical Treatise on Foundations.......................................8vo, 5 00
Rice's Concrete Block Manufacture.......................................8vo, 2 00
Richardson's Modern Asphalt Pavements.......................................8vo, 3 00
Richey's Handbook for Superintendents of Construction.......................................16mo, mor. 4 00
* Ries's Clays: Their Occurrence, Properties, and Uses.......................................8vo, 5 00
Sabin's Industrial and Artistic Technology of Paints and Varnish.......................................8vo, 3 00
* Schwarz's Longleaf Pine in Virgin Forest.......................................12mo, 1 25
Snow's Principal Species of Wood.......................................8vo, 3 50
Spalding's Hydraulic Cement.......................................12mo, 2 00
Text-book on Roads and Pavements.......................................12mo, 2 00
Taylor and Thompson's Treatise on Concrete, Plain and Reinforced.......................................8vo, 5 00
Thurston's Materials of Engineering. In Three Parts.......................................8vo, 8 00
Part I. Non-metallic Materials of Engineering and Metallurgy.......................................8vo, 2 00
Part II. Iron and Steel.......................................8vo, 3 50
Part III. A Treatise on Brasses, Bronzes, and Other Alloys and their Constituents.......................................8vo, 2 50
Tillson's Street Pavements and Paving Materials.......................................8vo, 4 00
Turneaure and Maurer's Principles of Reinforced Concrete Construction.......................................8vo, 3 00
Waterbury's Manual of Instructions for the Use of Students in Cement Laboratory Practice. (In Press.)

Wood's (De V.) Treatise on the Resistance of Materials, and an Appendix on the Preservation of Timber. 8vo, 2 00
Wood's (M. P.) Rustless Coatings: Corrosion and Electrolysis of Iron and Steel. 8vo, 4 00

RAILWAY ENGINEERING.

Andrews's Handbook for Street Railway Engineers ....3x5 inches, mor. 1 25
Berg's Buildings and Structures of American Railroads 4to, 5 00
Brooks's Handbook of Street Railroad Location. ....16mo, mor. 1 50
Butt's Civil Engineer's Field-book. ....16mo, mor. 2 50
Crandall's Railway and Other Earthwork Tables. 8vo, 1 50
Transition Curve. ....16mo, mor. 1 50
* Crockett's Methods for Earthwork Computations. 8vo, 1 50
Dawson's "Engineering" and Electric Traction Pocket-book. ....16mo, mor. 5 00
Dredge's History of the Pennsylvania Railroad: (1879). ....Paper, 5 00
Fisher's Table of Cubic Yards. ....Cardboard, 25
Godwin's Railroad Engineers' Field-book and Explorers' Guide... 16mo, mor. 2 50
Hudson's Tables for Calculating the Cubic Contents of Excavations and Embankments. 8vo, 1 00
Ives and Hilts's Problems in Surveying, Railroad Surveying and Geodesy 16mo, mor. 1 50
Molitor and Beard's Manual for Resident Engineers. 16mo, 1 00
Nagle's Field Manual for Railroad Engineers. ....16mo, mor. 3 00
Philbrick's Field Manual for Engineers. ....16mo, mor. 3 00
Raymond's Railroad Engineering. 3 volumes.
Vol. I. Railroad Field Geometry. (In Preparation.)
Vol. II. Elements of Railroad Engineering. 8vo, 3 50
Vol. III. Railroad Engineer's Field Book. (In Preparation.)
Searles's Field Engineering. ....16mo, mor. 3 00
Railroad Spiral. ....16mo, mor. 1 50
Taylor's Prismoidal Formulæ and Earthwork. 8vo, 1 50
* Trautwine's Field Practice of Laying Out Circular Curves for Railroads. 12mo. mor. 2 50
* Method of Calculating the Cubic Contents of Excavations and Embankments by the Aid of Diagrams. 8vo, 2 00
Webb's Economics of Railroad Construction. ....Large 12mo, 2 50
Railroad Construction. ....16mo, mor. 5 00
Wellington's Economic Theory of the Location of Railways. ....Small 8vo, 5 00

DRAWING.

Barr's Kinematics of Machinery. 8vo, 2 50
* Bartlett's Mechanical Drawing. 8vo, 3 00
* " " " Abridged Ed. 8vo, 1 50
Coolidge's Manual of Drawing. ....8vo, paper, 1 00
Coolidge and Freeman's Elements of General Drafting for Mechanical Engineers. ....Oblong 4to, 2 50
Durley's Kinematics of Machines. 8vo, 4 00
Emch's Introduction to Projective Geometry and its Applications. 8vo, 2 50
Hill's Text-book on Shades and Shadows, and Perspective. 8vo, 2 00
Jamison's Advanced Mechanical Drawing. 8vo, 2 00
Elements of Mechanical Drawing. 8vo, 2 50
Jones's Machine Design:
Part I. Kinematics of Machinery. 8vo, 1 50
Part II. Form, Strength, and Proportions of Parts. 8vo, 3 00
MacCord's Elements of Descriptive Geometry. 8vo, 3 00
Kinematics; or, Practical Mechanism. 8vo, 5 00
Mechanical Drawing. 4to, 4 00
Velocity Diagrams. 8vo, 1 50

McLeod's Descriptive Geometry..............................Large 12mo, 1 50
* Mahan's Descriptive Geometry and Stone-cutting....................8vo, 1 50
Industrial Drawing. (Thompson)..............................8vo, 3 50
Moyer's Descriptive Geometry...8vo, 2 00
Reed's Topographical Drawing and Sketching..........................4to, 5 00
Reid's Course in Mechanical Drawing.................................8vo, 2 00
Text-book of Mechanical Drawing and Elementary Machine Design.8vo, 3 00
Robinson's Principles of Mechanism..................................8vo, 3 00
Schwamb and Merrill's Elements of Mechanism........................8vo, 3 00
Smith's (R. S.) Manual of Topographical Drawing. (McMillan)........8vo, 2 50
Smith (A. W.) and Marx's Machine Design.............................8vo, 3 00
* Titsworth's Elements of Mechanical Drawing................Oblong 8vo, 1 25
Warren's Drafting Instruments and Operations.......................12mo, 1 25
Elements of Descriptive Geometry, Shadows, and Perspective.......8vo, 3 50
Elements of Machine Construction and Drawing.................8vo, 7 50
Elements of Plane and Solid Free-hand Geometrical Drawing.....12mo, 1 00
General Problems of Shades and Shadows.......................8vo, 3 00
Manual of Elementary Problems in the Linear Perspective of Form and Shadow...12mo, 1 00
Manual of Elementary Projection Drawing.....................12mo, 1 50
Plane Problems in Elementary Geometry.......................12mo, 1 25
Problems, Theorems, and Examples in Descriptive Geometry.......8vo, 2 50
Weisbach's Kinematics and Power of Transmission. (Hermann and Klein)...8vo, 5 00
Wilson's (H. M.) Topographic Surveying..............................8vo, 3 50
Wilson's (V. T.) Free-hand Lettering................................8vo, 1 00
Free-hand Perspective.......................................8vo, 2 50
Woolf's Elementary Course in Descriptive Geometry.............Large 8vo, 3 00

ELECTRICITY AND PHYSICS.

* Abegg's Theory of Electrolytic Dissociation. (von Ende)..........12mo, 1 25
Andrews's Hand-Book for Street Railway Engineering ....3×5 inches, mor. 1 25
Anthony and Brackett's Text-book of Physics. (Magie)........Large 12mo, 3 00
Anthony's Lecture-notes on the Theory of Electrical Measurements....12mo, 1 00
Benjamin's History of Electricity...................................8vo, 3 00
Voltaic Cell..8vo, 3 00
Betts's Lead Refining and Electrolysis..............................8vo, 4 00
Classen's Quantitative Chemical Analysis by Electrolysis. (Boltwood)..8vo, 3 00
* Collins's Manual of Wireless Telegraphy..........................12mo, 1 50
Mor. 2 00
Crehore and Squier's Polarizing Photo-chronograph...................8vo, 3 00
* Danneel's Electrochemistry. (Merriam).............................12mo, 1 25
Dawson's "Engineering" and Electric Traction Pocket-book....16mo, mor. 5 00
Dolezalek's Theory of the Lead Accumulator (Storage Battery). (von Ende) 12mo, 2 50
Duhem's Thermodynamics and Chemistry. (Burgess)....................8vo, 4 00
Flather's Dynamometers, and the Measurement of Power...............12mo, 3 00
Gilbert's De Magnete. (Mottelay)....................................8vo, 2 50
* Hanchett's Alternating Currents..................................12mo, 1 00
Hering's Ready Reference Tables (Conversion Factors)..........16mo, mor. 2 50
* Hobart and Ellis's High-speed Dynamo Electric Machinery...........8vo, 6 00
Holman's Precision of Measurements..................................8vo, 2 00
Telescopic Mirror-scale Method, Adjustments, and Tests....Large 8vo, 75
* Karapetoff's Experimental Electrical Engineering....................8vo, 6 00
Kinzbrunner's Testing of Continuous-current Machines................8vo, 2 00
Landauer's Spectrum Analysis. (Tingle)..............................8vo, 3 00
Le Chatelier's High-temperature Measurements. (Boudouard—Burgess)..12mo, 3 00
Löb's Electrochemistry of Organic Compounds. (Lorenz)..............8vo, 3 00
* London's Development and Electrical Distribntion of Water Fower....8vo, 3 00

* Lyons's Treatise on Electromagnetic Phenomena. Vols. I. and II. 8vo, each, 6 00
* Michie's Elements of Wave Motion Relating to Sound and Light. 8vo, 4 00
Morgan's Outline of the Theory of Solution and its Results. 12mo, 1 00
* Physical Chemistry for Electrical Engineers. 12mo, 1 50
Niaudet's Elementary Treatise on Electric Batteries. (Fishback). . . .12mo, 2 50
* Norris's Introduction to the Study of Electrical Engineering. 8vo, 2 50
* Parshall and Hobart's Electric Machine Design. . . . . .4to, half mor. 12 50
Reagan's Locomotives: Simple, Compound, and Electric. New Edition.
Large 12mo, 3 50
* Rosenberg's Electrical Engineering. (Haldane Gee—Kinzbrunner). . . .8vo, 2 00
Ryan, Norris, and Hoxie's Electrical Machinery. Vol. I. 8vo, 2 50
Schapper's Laboratory Guide for Students in Physical Chemistry 12mo, 1 00
* Tillman's Elementary Lessons in Heat. 8vo, 1 50
Tory and Pitcher's Manual of Laboratory Physics. . . . . .Large 12mo, 2 00
Ulke's Modern Electrolytic Copper Refining. 8vo, 3 00

LAW.

* Davis's Elements of Law. 8vo, 2 50
* Treatise on the Military Law of United States. 8vo, 7 00
* Sheep, 7 50
* Dudley's Military Law and the Procedure of Courts-martial . . . .Large 12mo, 2 50
Manual for Courts-martial. . . . . .16mo, mor. 1 50
Wait's Engineering and Architectural Jurisprudence. 8vo, 6 00
Sheep, 6 50
Law of Contracts. 8vo, 3 00
Law of Operations Preliminary to Construction in Engineering and Architecture. 8vo 5 00
Sheep, 5 50

MATHEMATICS.

Baker's Elliptic Functions 8vo, 1 50
Briggs's Elements of Plane Analytic Geometry. (Bôcher). 12mo, 1 00
*Buchanan's Plane and Spherical Trigonometry. 8vo, 1 00
Byerley's Harmonic Functions. 8vo, 1 00
Chandler's Elements of the Infinitesimal Calculus. 12mo, 2 00
Compton's Manual of Logarithmic Computations 12mo, 1 50
* Dickson's College Algebra. . . . . .Large 12mo, 1 50
* Introduction to the Theory of Algebraic Equations . . . . .Large 12mo, 1 25
Emch's Introduction to Projective Geometry and its Applications 8vo, 2 50
Fiske's Functions of a Complex Variable. 8vo, 1 00
Halsted's Elementary Synthetic Geometry 8vo, 1 50
Elements of Geometry 8vo, 1 75
* Rational Geometry 12mo, 1 50
Hyde's Grassmann's Space Analysis. 8vo, 1 00
* Jonnson's (*J. B.*) Three-place Logarithmic Tables: Vest-pocket size, paper, 15
100 copies, 5 00
* Mounted on heavy cardboard, 8 × 10 inches, 25
10 copies, 2 00
Johnson's (W. W.) Abridged Editions of Differential and Integral Calculus
Large 12mo, 1 vol. 2 50
Curve Tracing in Cartesian Co-ordinates 12mo, 1 00
Differential Equations. 8vo, 1 00
Elementary Treatise on Differential Calculus . . . . .Large 12mo, 1 50
Elementary Treatise on the Integral Calculus . . . . .Large 12mo, 1 50
* Theoretical Mechanics. 12mo, 3 00
Theory of Errors and the Method of Least Squares. 12mo, 1 50
Treatise on Differential Calculus. . . . . .Large 12mo, 3 00
Treatise on the Integral Calculus. . . . . .Large 12mo, 3 00
Treatise on Ordinary and Partial Differential Equations. .Large 12mo, 3 50

Laplace's Philosophical Essay on Probabilities. (Truscott and Emory)..12mo, 2 00
* Ludlow and Bass's Elements of Trigonometry and Logarithmic and Other Tables 8vo, 3 00
Trigonometry and Tables published separately ....Each, 2 00
* Ludlow's Logarithmic and Trigonometric Tables 8vo, 1 00
Macfarlane's Vector Analysis and Quaternions......................8vo, 1 00
McMahon's Hyperbolic Functions.....................................8vo, 1 00
Manning's Irrational Numbers and their Representation by Sequences and Series 12mo, 1 25
Mathematical Monographs. Edited by Mansfield Merriman and Robert S. Woodward.................................Octavo, each 1 00

No. 1. History of Modern Mathematics, by David Eugene Smith. No. 2. Synthetic Projective Geometry, by George Bruce Halsted. No. 3. Determinants, by Laenas Gifford Weld. No. 4. Hyperbolic Functions, by James McMahon. No. 5. Harmonic Functions, by William E. Byerly. No. 6. Grassmann's Space Analysis, by Edward W. Hyde. No. 7. Probability and Theory of Errors, by Robert S. Woodward. No. 8. Vector Analysis and Quaternions, by Alexander Macfarlane. No. 9. Differential Equations, by William Woolsey Johnson. No. 10. The Solution of Equations, by Mansfield Merriman. No. 11. Functions of a Complex Variable, by Thomas S. Fiske.

Maurer's Technical Mechanics.......................................8vo, 4 00
Merriman's Method of Least Squares...............................8vo, 2 00
Solution of Equations 8vo, 1 00
Rice and Johnson's Differential and Integral Calculus. 2 vols. in one. Large 12mo, 1 50
Elementary Treatise on the Differential Calculus...........Large 12mo, 3 00
Smith's History of Modern Mathematics 8vo, 1 00
* Veblen and Lennes's Introduction to the Real Infinitesimal Analysis of One Variable..8vo, 2 00
* Waterbury's Vest Pocket Hand-Book of Mathematics for Engineers. $2\frac{7}{8}\times5\frac{3}{8}$ inches, mor. 1 00
Weld's Determinations..8vo, 1 00
Wood's Elements of Co-ordinate Geometry...........................8vo, 2 00
Woodward's Probability and Theory of Errors........................8vo, 1 00

MECHANICAL ENGINEERING.

MATERIALS OF ENGINEERING, STEAM-ENGINES AND BOILERS.

Bacon's Forge Practice..12mo, 1 50
Baldwin's Steam Heating for Buildings.............................12mo, 2 50
Barr's Kinematics of Machinery.....................................8vo, 2 50
* Bartlett's Mechanical Drawing.....................................8vo, 3 00
* " " " Abridged Ed.........................8vo, 1 50
Benjamin's Wrinkles and Recipes...................................12mo, 2 00
* Burr's Ancient and Modern Engineering and the Isthmian Canal......8vo, 3 50
Carpenter's Experimental Engineering...............................8vo, 6 00
Heating and Ventilating Buildings..................................8vo, 4 00
Clerk's Gas and Oil Engine..............................Large 12mo, 4 00
Compton's First Lessons in Metal Working 12mo, 1 50
Compton and De Groodt's Speed Lathe...............................12mo, 1 50
Coolidge's Manual of Drawing...............................8vo, paper, 1 00
Coolidge and Freeman's Elements of General Drafting for Mechanical Engineers...Oblong 4to, 2 50
Cromwell's Treatise on Belts and Pulleys..........................12mo, 1 50
Treatise on Toothed Gearing.......................................12mo, 1 50
Durley's Kinematics of Machines....................................8vo, 4 00

Flather's Dynamometers and the Measurement of Power............12mo, 3 00
Rope Driving..12mo, 2 00
Gill's Gas and Fuel Analysis for Engineers........................12mo, 1 25
Goss's Locomotive Sparks...8vo, 2 00
Greene's Pumping Machinery. (In Preparation.)
Hering's Ready Reference Tables (Conversion Factors)..........16mo, mor. 2 50
* Hobart and Ellis's High Speed Dynamo Electric Machinery..........8vo. 6 00
Hutton's Gas Engine...8vo, 5 00
Jamison's Advanced Mechanical Drawing............................8vo, 2 00
Elements of Mechanical Drawing..............................8vo, 2 50
Jones's Machine Design:
Part I. Kinematics of Machinery..............................8vo, 1 50
Part II. Form, Strength, and Proportions of Parts..............8vo, 3 00
Kent's Mechanical Engineers' Pocket-book....................16mo, mor. 5 00
Kerr's Power and Power Transmission..............................8vo, 2 00
Leonard's Machine Shop Tools and Methods.........................8vo, 4 00
* Lorenz's Modern Refrigerating Machinery. (Pope, Haven, and Dean)...8vo, 4 00
MacCord's Kinematics; or, Practical Mechanism.....................8vo, 5 00
Mechanical Drawing..4to, 4 00
Velocity Diagrams...8vo, 1 50
MacFarland's Standard Reduction Factors for Gases..................8vo, 1 50
Mahan's Industrial Drawing. (Thompson)...........................8vo, 3 50
* Parshall and Hobart's Electric Machine Design.....Small 4to, half leather, 12 50
Peele's Compressed Air Plant for Mines.............................8vo, 3 00
Poole's Calorific Power of Fuels...................................8vo, 3 00
* Porter's Engineering Reminiscences, 1855 to 1882....................8vo, 3 00
Reid's Course in Mechanical Drawing...............................8vo, 2 00
Text-book of Mechanical Drawing and Elementary Machine Design.8vo, 3 00
Richard's Compressed Air...12mo, 1 50
Robinson's Principles of Mechanism................................8vo, 3 00
Schwamb and Merrill's Elements of Mechanism.......................8vo, 3 00
Smith's (O.) Press-working of Metals..............................8vo, 3 00
Smith (A. W.) and Marx's Machine Design...........................8vo, 3 00
Sorel's Carbureting and Combustion in Alcohol Engines. (Woodward and Preston). Large 12mo, 3 00
Thurston's Animal as a Machine and Prime Motor, and the Laws of Energetics. 12mo, 1 00
Treatise on Friction and Lost Work in Machinery and Mill Work... 8vo, 3 00
Tillson's Complete Automobile Instructor..........................16mo, 1 50
mor. 2 00
* Titsworth's Elements of Mechanical Drawing.................Oblong 8vo, 1 25
Warren's Elements of Machine Construction and Drawing.............8vo, 7 50
* Waterbury's Vest Pocket Hand Book of Mathematics for Engineers. $2\frac{7}{8}\times5\frac{3}{8}$ inches, mor. 1 00
Weisbach's Kinematics and the Power of Transmission. (Herrmann—Klein)..8vo, 5 00
Machinery of Transmission and Governors. (Herrmann—Klein)...8vo, 5 00
Wood's Turbines...8vo, 2 50

MATERIALS OF ENGINEERING.

* Bovey's Strength of Materials and Theory of Structures............. 8vo, 7 50
Burr's Elasticity and Resistance of the Materials of Engineering.........8vo, 7 50
Church's Mechanics of Engineering.................................8vo, 6 00
* Greene's Structural Mechanics....................................8vo, 2 50
Holley and Ladd's Analysis of Mixed Paints, Color Pigments, and Varnishes. Large 12mo, 2 50
Johnson's Materials of Construction................................8vo, 6 00
Keep's Cast Iron...8vo, 2 50
Lanza's Applied Mechanics...8vo, 7 50

Maire's Modern Pigments and their Vehicles..............................12mo, 2 00
Martens's Handbook on Testing Materials. (Henning)................8vo, 7 50
Maurer's Technical Mechanics......................................8vo, 4 00
Merriman's Mechanics of Materials.................................8vo, 5 00
* Strength of Materials..12mo, 1 00
Metcalf's Steel. A Manual for Steel-users.........................12mo, 2 00
Sabin's Industrial and Artistic Technology of Paints and Varnish........8vo, 3 00
Smith's Materials of Machines.....................................12mo, 1 00
Thurston's Materials of Engineering........................3 vols., 8vo, 8 00
Part I. Non-metallic Materials of Engineering and Metallurgy...8vo, 2 00
Part II. Iron and Steel...8vo, 3 50
Part III. A Treatise on Brasses, Bronzes, and Other Alloys and their Constituents..8vo, 2 50
Wood's (De V.) Elements of Analytical Mechanics...................8vo, 3 00
Treatise on the Resistance of Materials and an Appendix on the Preservation of Timber..8vo, 2 00
Wood's (M. P.) Rustless Coatings: Corrosion and Electrolysis of Iron and Steel..8vo, 4 00

STEAM-ENGINES AND BOILERS.

Berry's Temperature-entropy Diagram...............................12mo, 1 25
Carnot's Reflections on the Motive Power of Heat. (Thurston).......12mo, 1 50
Chase's Art of Pattern Making.....................................12mo, 2 50
Creighton's Steam-engine and other Heat-motors....................8vo, 5 00
Dawson's "Engineering" and Electric Traction Pocket-book.....16mo, mor. 5 00
Ford's Boiler Making for Boiler Makers............................18mo, 1 00
Gebhardt's Steam Power Plant Engineering. (In Press.)
Goss's Locomotive Performance.....................................8vo, 5 00
Hemenway's Indicator Practice and Steam-engine Economy..........12mo, 2 00
Hutton's Heat and Heat-engines....................................8vo, 5 00
Mechanical Engineering of Power Plants............................8vo, 5 00
Kent's Steam boiler Economy.......................................8vo, 4 00
Kneass's Practice and Theory of the Injector......................8vo, 1 50
MacCord's Slide-valves..8vo, 2 00
Meyer's Modern Locomotive Construction............................4to, 10 00
Moyer's Steam Turbines. (In Press.)
Peabody's Manual of the Steam-engine Indicator....................12mo, 1 50
Tables of the Properties of Saturated Steam and Other Vapors......8vo, 1 00
Thermodynamics of the Steam-engine and Other Heat-engines......8vo, 5 00
Valve-gears for Steam-engines.....................................8vo, 2 50
Peabody and Miller's Steam-boilers................................8vo, 4 00
Pray's Twenty Years with the Indicator.......................Large 8vo, 2 50
Pupin's Thermodynamics of Reversible Cycles in Gases and Saturated Vapors. (Osterberg)..12mo, 1 25
Reagan's Locomotives: Simple, Compound, and Electric. New Edition. Large 12mo, 3 50
Sinclair's Locomotive Engine Running and Management.............12mo, 2 00
Smart's Handbook of Engineering Laboratory Practice...............12mo, 2 50
Snow's Steam-boiler Practice......................................8vo, 3 00
Spangler's Notes on Thermodynamics................................12mo, 1 00
Valve-gears...8vo, 2 50
Spangler, Greene, and Marshall's Elements o Steam-engineering.......8vo, 3 00
Thomas's Steam-turbines...8vo, 4 00
Thurston's Handbook of Engine and Boiler Trials, and the Use of the Indicator and the Prony Brake....................................8vo, 5 00
Handy Tables..8vo, 1 50
Manual of Steam-boilers, their Designs, Construction, and Operation..8vo, 5 00

Thurston's Manual of the Steam-engine......................2 vols., 8vo, 10 00
Part I. History, Structure, and Theory......................8vo, 6 00
Part II. Design, Construction, and Operation................8vo, 6 00
Steam-boiler Explosions in Theory and in Practice............12mo, 1 50
Wehrenfenning's Analysis and Softening of Boiler Feed-water (Patterson) 8vo, 4 00
Weisbach's Heat, Steam, and Steam-engines. (Du Bois)..............8vo, 5 00
Whitham's Steam-engine Design..................................8vo, 5 00
Wood's Thermodynamics, Heat Motors, and Refrigerating Machines...8vo, 4 00

MECHANICS PURE AND APPLIED.

Church's Mechanics of Engineering...............................8vo, 6 00
Notes and Examples in Mechanics...............................8vo, 2 00
Dana's Text-book of Elementary Mechanics for Colleges and Schools..12mo, 1 50
Du Bois's Elementary Principles of Mechanics:
Vol. I. Kinematics...8vo, 3 50
Vol. II. Statics...8vo, 4 00
Mechanics of Engineering. Vol. I.........................Small 4to, 7 50
Vol. II.........................Small 4to, 10 00
* Greene's Structural Mechanics...................................8vo, 2 50
James's Kinematics of a Point and the Rational Mechanics of a Particle.
Large 12mo, 2 00
* Johnson's (W. W.) Theoretical Mechanics........................12mo, 3 00
Lanza's Applied Mechanics..8vo, 7 50
* Martin's Text Book on Mechanics, Vol. I, Statics..................12mo, 1 25
* Vol. 2, Kinematics and Kinetics ..12mo, 1 50
Maurer's Technical Mechanics.....................................8vo, 4 00
* Merriman's Elements of Mechanics...............................12mo, 1 00
Mechanics of Materials.......................................8vo, 5 00
* Michie's Elements of Analytical Mechanics........................8vo, 4 00
Robinson's Principles of Mechanism...............................8vo, 3 00
Sanborn's Mechanics Problems..............................Large 12mo, 1 50
Schwamb and Merrill's Elements of Mechanism.......................8vo, 3 00
Wood's Elements of Analytical Mechanics...........................8vo, 3 00
Principles of Elementary Mechanics...........................12mo, 1 25

MEDICAL.

* Abderhalden's Physiological Chemistry in Thirty Lectures. (Hall and Defren)
8vo, 5 00
von Behring's Suppression of Tuberculosis. (Bolduan)..............12mo, 1 00
* Bolduan's Immune Sera..12mo, 1 50
Davenport's Statistical Methods with Special Reference to Biological Variations..16mo, mor. 1 50
Ehrlich's Collected Studies on Immunity. (Bolduan).................8vo, 6 00
* Fischer's Physiology of Alimentation.................Large 12mo, cloth, 2 00
de Fursac's Manual of Psychiatry. (Rosanoff and Collins).......Large 12mo, 2 50
Hammarsten's Text-book on Physiological Chemistry. (Mandel)........8vo, 4 00
Jackson's Directions for Laboratory Work in Physiological Chemistry...8vo, 1 25
Lassar-Cohn's Practical Urinary Analysis. (Lorenz)................12mo, 1 00
Mandel's Hand Book for the Bio-Chemical Laboratory................12mo, 1 50
* Pauli's Physical Chemistry in the Service of Medicine. (Fischer).....12mo, 1 25
* Pozzi-Escot's Toxins and Venoms and their Antibodies. (Cohn).......12mo, 1 00
Rostoski's Serum Diagnosis. (Bolduan)............................12mo, 1 00
Ruddiman's Incompatibilities in Prescriptions........................8vo, 2 00
Whys in Pharmacy..12mo, 1 00
Salkowski's Physiological and Pathological Chemistry. (Orndorff)......8vo, 2 50
* Satterlee's Outlines of Human Embryology........................12mo, 1 25
Smith's Lecture Notes on Chemistry for Dental Students................8vo, 2 50

Steel's Treatise on the Diseases of the Dog..........................8vo, 3 50
* Whipple's Typhoid Fever.................................Large 12mo, 3 00
Woodhull's Notes on Military Hygiene 16mo, 1 50
* Personal Hygiene..12mo, 1 00
Worcester and Atkinson's Small Hospitals Establishment and Maintenance, and S ggestions for Hospital Architecture, with Plans for a Small Hospital 12mo, 1 25

METALLURGY.

Betts's Lead Refining by Electrolysis 8vo, 4 00
Bolland's Encyclopedia of Founding and Dictionary of Foundry Terms Used in the Practice of Moulding 12mo, 3 00
Iron Founder 12mo, 2 50
" " Supplement 12mo, 2 50
Douglas's Untechnical Addresses on Technical Subjects..............12mo, 1 00
Goesel's Minerals and Metals: A Reference Book ....16mo, mor. 3 00
* Iles's Lead-smelting 12mo, 2 50
Keep's Cast Iron 8vo, 2 50
Le Chatelier's High-temperature Measurements. (Boudouard—Burgess) 12mo, 3 00
Metcalf's Steel. A Manual for Steel-users 12mo, 2 00
Miller's Cyanide Process 12mo, 1 00
Minet's Production of Aluminium and its Industrial Use. (Waldo) ...12mo, 2 50
Robine and Lenglen's Cyanide Industry. (Le Clerc) 8vo, 4 00
Ruer's Elements of Metallography. (Mathewson) (In Press.)
Smith's Materials of Machines 12mo, 1 00
Thurston's Materials of Engineering. In Three Parts 8vo, 8 00
Part I. Non-metallic Materials of Engineering and Metallurgy...8vo, 2 00
Part II. Iron and Steel..8vo, 3 50
Part III. A Treatise on Brasses, Bronzes, and Other Alloys and their Constituents..8vo, 2 50
Ulke's Modern Electrolytic Copper Refining 8vo, 3 00
West's American Foundry Practice 12mo, 2 50
Moulder's Text Book 12mo, 2 50
Wilson's Chlorination Process 12mo, 1 50
Cyanide Processes..12mo, 1 50

MINERALOGY.

Barringer's Description of Minerals of Commercial Value......Oblong, mor. 2 50
Boyd's Resources of Southwest Virginia.............................8vo, 3 00
Boyd's Map of Southwest Virginia.................................Pocket-book form. 2 00
* Browning's Introduction to the Rarer Elements.......................8vo, 1 50
Brush's Manual of Determinative Mineralogy. (Penfield)...............8vo, 4 00
Butler's Pocket Hand-Book of Minerals.........................16mo, mor. 3 00
Chester's Catalogue of Minerals...............................8vo, paper, 1 00
Cloth, 1 25
* Crane's Gold and Silver. 8vo, 5 00
Dana's First Appendix to Dana's New "System of Mineralogy.."..Large 8vo, 1 00
Manual of Mineralogy and Petrography........................12mo 2 00
Minerals and How to Study Them 12mo, 1 50
System of Mineralogy.......................Large 8vo, half leather, 12 50
Text-book of Mineralogy..8vo, 4 00
Douglas's Untechnical Addresses on Technical Subjects..............12mo, 1 00
Eakle's Mineral Tables..8vo, 1 25
Stone and Clay Products Used in Engineering. (In Preparation.)
Egleston's Catalogue of Minerals and Synonyms.......................8vo, 2 50
Goesel's Minerals and Metals: A Reference Book..............16mo, mor. 3 00
Groth's Introduction to Chemical Crystallography (Marshall)........ 12mo, 1 25

* Iddings's Rock Minerals	8vo,	5 00
Johannsen's Determination of Rock-forming Minerals in Thin Sections	8vo,	4 00
* Martin's Laboratory Guide to Qualitative Analysis with the Blowpipe	12mo,	60
Merrill's Non-metallic Minerals: Their Occurrence and Uses	8vo,	4 00
Stones for Building and Decoration	8vo,	5 00
* Penfield's Notes on Determinative Mineralogy and Record of Mineral Tests.	8vo, paper,	50
Tables of Minerals, Including the Use of Minerals and Statistics of Domestic Production	8vo,	1 00
* Pirsson's Rocks and Rock Minerals	12mo,	2 50
* Richards's Synopsis of Mineral Characters	12mo, mor.	1 25
* Ries's Clays: Their Occurrence, Properties, and Uses	8vo,	5 00
* Tillman's Text-book of Important Minerals and Rocks	8vo,	2 00

MINING.

* Beard's Mine Gases and Explosions	Large 12mo,	3 00
Boyd's Map of Southwest Virginia	Pocket-book form	2 00
Resources of Southwest Virginia	8vo,	3 00
* Crane's Gold and Silver	8vo,	5 00
Douglas's Untechnical Addresses on Technical Subjects	12mo	1 00
Eissler's Modern High Explosives	8vo,	4 00
Goesel's Minerals and Metals: A Reference Book	16mo, mor.	3 00
Ihlseng's Manual of Mining	8vo,	5 00
* Iles's Lead-smelting	12mo,	2 50
Miller's Cyanide Process	12mo,	1 00
O'Driscoll's Notes on the Treatment of Gold Ores	8vo,	2 00
Peele's Compressed Air Plant for Mines	8vo,	3 00
Riemer's Shaft Sinking Under Difficult Conditions. (Corning and Peele)	8vo,	3 00
Robine and Lenglen's Cyanide Industry. (Le Clerc)	8vo,	4 00
* Weaver's Military Explosives	8vo,	3 00
Wilson's Chlorination Process	12mo,	1 50
Cyanide Processes	12mo,	1 50
Hydraulic and Placer Mining. 2d edition, rewritten	12mo,	2 50
Treatise on Practical and Theoretical Mine Ventilation	12mo,	1 25

SANITARY SCIENCE.

Association of State and National Food and Dairy Departments, Hartford Meeting, 1906	8vo,	3 00
Jamestown Meeting, 1907	8vo,	3 00
* Bashore's Outlines of Practical Sanitation	12mo,	1 25
Sanitation of a Country House	12mo,	1 00
Sanitation of Recreation Camps and Parks	12mo,	1 00
Folwell's Sewerage. (Designing, Construction, and Maintenance)	8vo,	3 00
Water-supply Engineering	8vo,	4 00
Fowler's Sewage Works Analyses	12mo,	2 00
Fuertes's Water-filtration Works	12mo,	2 50
Water and Public Health	12mo,	1 50
Gerhard's Guide to Sanitary House-inspection	16mo,	1 00
* Modern Baths and Bath Houses	8vo,	3 00
Sanitation of Public Buildings	12mo,	1 50
Hazen's Clean Water and How to Get It	Large 12mo,	1 50
Filtration of Public Water-supplies	8vo,	3 00
Kinnicut, Winslow and Pratt's Purification of Sewage. (In Press.)		
Leach's Inspection and Analysis of Food with Special Reference to State Control	8vo,	7 00
Mason's Examination of Water. (Chemical and Bacteriological)	12mo,	1 25
Water-supply. (Considered Principally from a Sanitary Standpoint)	8vo,	4 00

* Merriman's Elements of Sanitary Engineering........................8vo,	2 00
Ogden's Sewer Design...12mo,	2 00
Parsons's Disposal of Municipal Refuse................................8vo,	2 00
Prescott and Winslow's Elements of Water Bacteriology, with Special Reference to Sanitary Water Analysis..........................12mo,	1 50
* Price's Handbook on Sanitation.................................12mo,	1 50
Richards's Cost of Food. A Study in Dietaries........................12mo,	1 00
Cost of Living as Modified by Sanitary Science...................12mo,	1 00
Cost of Shelter..12mo,	1 00
* Richards and Williams's Dietary Computer..........................8vo,	1 50
Richards and Woodman's Air, Water, and Food from a Sanitary Standpoint..8vo,	2 00
Rideal's Disinfection and the Preservation of Food.....................8vo,	4 00
Sewage and Bacterial Purification of Sewage...................8vo,	4 00
Soper's Air and Ventilation of Subways.......................Large 12mo,	2 50
Turneaure and Russell's Public Water-supplies........................8vo,	5 00
Venable's Garbage Crematories in America............................8vo,	2 00
Method and Devices for Bacterial Treatment of Sewage............8vo,	3 00
Ward and Whipple's Freshwater Biology. (In Press.)	
Whipple's Microscopy of Drinking-water..............................8vo,	3 50
* Typhod Fever......................................Large 12mo,	3 00
Value of Pure Water.................................Large 12mo,	1 00
Winslow's Bacterial Classification. (In Press.)	
Winton's Microscopy of Vegetable Foods.............................8vo,	7 50

MISCELLANEOUS.

Emmons's Geological Guide-book of the Rocky Mountain Excursion of the International Congress of Geologists...................Large 8vo,	1 50
Ferrel's Popular Treatise on the Winds..............................8vo,	4 00
Fitzgerald's Boston Machinist.....................................18mo,	1 00
Gannett's Statistical Abstract of the World..........................24mo,	75
Haines's American Railway Management............................12mo,	2 50
* Hanusek's The Microscopy of Technical Products. (Winton)...........8vo,	5 00
Ricketts's History of Rensselaer Polytechnic Institute 1824–1894. Large 12mo,	3 00
Rotherham's Emphasized New Testament.....................Large 8vo,	2 00
Standage's Decoration of Wood, Glass, Metal, etc...................12mo,	2 00
Thome's Structural and Physiological Botany. (Bennett).............16mo,	2 25
Westermaier's Compendium of General Botany. (Schneider)............8vo,	2 00
Winslow's Elements of Applied Microscopy.........................12mo,	1 50

HEBREW AND CHALDEE TEXT-BOOKS.

Green's Elementary Hebrew Grammar.............................12mo,	1 25
Gesenius's Hebrew and Chaldee Lexicon to the Old Testament Scriptures. (Tregelles).....................................Small 4to, half mor.	5 00